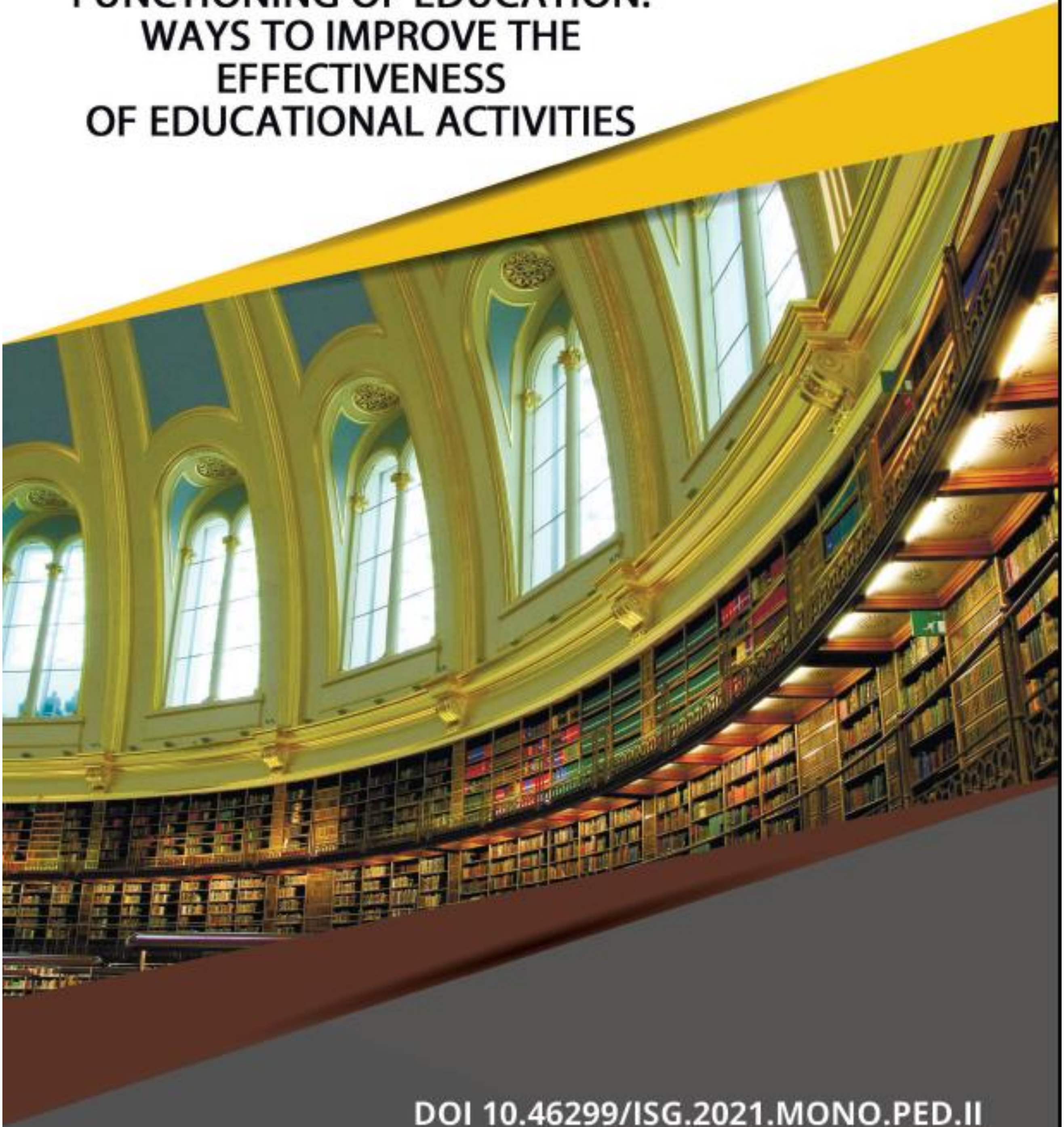
MONOGRAPH

THEORETICAL FOUNDATIONS OF THE FUNCTIONING OF EDUCATION. WAYS TO IMPROVE THE EFFECTIVENESS OF EDUCATIONAL ACTIVITIES

INTERNATIONAL SCIENCE GROUP



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Theoretical foundations of the functioning of Education. Ways to improve the effectiveness of educational activities

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6.4 Software package mathlab in linear algebra teaching

Using of mathematics software packages is one of indicators of the professional learning level of university graduates in information computer technologies (ICT). The problems of linear algebra are easy to algorithmize. To increase the efficiency of students' professional competencies formation, it makes sense to talk about algorithms in the form they would be performed by a computer. The ICT use in linear algebra teaching is becoming relevant. The main types of educational software are electronic textbooks, mathematics software packages and monitoring software. Electronic textbooks are used at lectures. There are textbooks with built-in knowledge control tools. Some textbooks contain practical tasks. However, there is no educational virtual environment for practical tasks. Mathematics software packages Mathematica, Maple, MathCAD, Matlab are used at practical classes. They will automate arithmetic calculations and enable students to focus on the essence of method. In some mathematical packages, including Matlab, tools for the distance development of a Web application focused on complex calculations were developed.

Monitoring software diagnoses checks and evaluates the knowledge, abilities and skills of the student. The linear testing algorithm is the simplest. For each student, a test variant is generated in a random order from a task repository. At linear and randomized testing, all participants have test questions of the same complexity. For example, the Google Forms service is widely available; it allows randomizing responses and uploading data in Excel format.

The programming language Matlab was developed by Cleve Moler, the Dean of Computer Science Department of New Mexico University in the late 1970s (Wikipedia). John Little together with Cleve Moler and Steve Bangert rewrote Matlab in C programming language and founded The MathWorks. At the moment, Matlab is a high-level language and interactive environment for developing algorithms, calculations, visualization and data analysis. It was originally developed for control systems designing. Matlab has quickly gained popularity in many scientific and engineering areas. The Matlab package is effective in the educational process, in particular, linear algebra teaching. In 2004, the Ministry of Education and Science of Ukraine decided to implement it the senior classes and universities. According to the UNESCO ICT Competency System, teachers who have sufficient competencies in ICT using in their professional practice will be able to ensure a high level of educational quality and effectively promote the students' ICT competences development (Terms of use – CC BY SA).

The Matlab environment use in diploma theses and dissertations will increase the complexity and importance of research. It will significantly redistribute the workload of teachers from homework tasks assessment to network project management in the Matlab environment. For example, the Eindhoven Technical University in the Netherlands (Technische Universiteit Eindhoven), most parts of scientific and teaching equipment are designed by students and produced within the educational process. So, it saves funds for the purchase of laboratory equipment.

Matlab is one of the ICT student's educational tools. On May 16, 2019, the First International Scientific and Practical Conference "Matlab and Computer Computing in Education, Science and Engineering" were held at the Kyiv National Aviation University.

One of the key priorities of UNESCO in the education field is to assist UNESCO Member States in developing a strategy and implementing a policy for informatization of education: national education systems modernizing through the introduction of innovative models of management of educational institutions, as well as a strategy for solving problems of improving the quality of education using of modern ICT and advanced pedagogical methods into everyday teaching practice. The IT development and implementation in education is constantly being researched by scientists of international organizations: UNESCO, UN, European Union, Council of Europe and others. The works of S. Peipert, M. Reznik, E. Patarakin, E. Polat, A. Khutorsky, B. Yarmakhov, A. Yastrebtsev, V. Bykov, N. Zhaldak, N. Zgurovsky, V. Kukharenko, V. Lapinsky, N. Morze, A. Pilipchuk, S. Rakova, S. Semerikov, E. Slovak, A. Stryuk, M. Shishkina and other researchers are devoted to this issue.

The purpose of paper is to present the capabilities of the Matlab package in linear algebra teaching using practical tasks.

Typical tasks of linear algebra are: finding a solution of linear equation systems, calculating the values of determinants and the matrix rank, finding a linear operator matrix, studying the roots of polynomials in one variable (Robert A Beezer, 2006). Linear algebra tasks are easy to algorithmize, but their implementation in the mathematical package Matlab is possible.

Let's consider matrices. Matrix elements are given in square brackets. Elements of one line are separated from each other by a space or comma. Lines are separated by a semicolon. To refer to a single matrix element, the matrix name and element indices in parentheses should be specified.

Basic operations with matrices:

- A + B is addition of matrices A and B (operation is possible with the same order of the matrices).
- A - B is subtracting of matrices A and B (operation is possible with the same order of matrices).
- A * B is production of matrices (an operation is possible when the number of columns of the matrix A is equal to the number of rows of the matrix B. The operation is not commutative).
- A * In the element wise multiplication of the matrices A and B of the same dimension.
- A. / B element wise division of the matrices A and B of the same dimension.
- A ^ k element wise raising of an array to k degree.
- A 'transposition of matrix A.
- A ^ –1 (or inv (A)) calculation of the inverse matrix A – 1 (the inverse matrix exists for square matrices whose determinant is nonzero).
- det (A) calculating of the matrix A determinant.
- size (A) definition of the matrix A dimension.
- trace (A) the sum of the elements on the main diagonal.

- sum (A) the sum of the elements in each matrix A column.
- prod (A) is the product of the elements in each column of the matrix A.
- diag (A) the column vector of the elements of the matrix A main diagonal.
- sort (A) sorting of matrix A each column.
- max (A) calculating the maximum in matrix A each column.
- min (A) calculation of the minimum in matrix A each column.
- mean (A) calculation of the average value in matrix A each column.
- rot90 (A) rotation of matrix A to the left by 90.
- fliplr (A) flip of matrix A from left to right.
- flipud (A) flip of matrix A from top to bottom.
- A (n, :) = [] deletion of the row number n from matrix A.

Let's consider the example. Matrices are given

$$A = \begin{pmatrix} -3 & 5 & 13 \\ 11 & -5 & 7 \\ 2 & -1 & -6 \end{pmatrix}, B = \begin{pmatrix} 2 & 9 & -8 \\ 1 & -15 & 14 \\ 4 & -12 & 3 \end{pmatrix}$$

Calculate

A+B, A-B, A·B, B·A, |A|, |B|, A^{-1}, B^{-1}. Set the matrix in the command window of the program

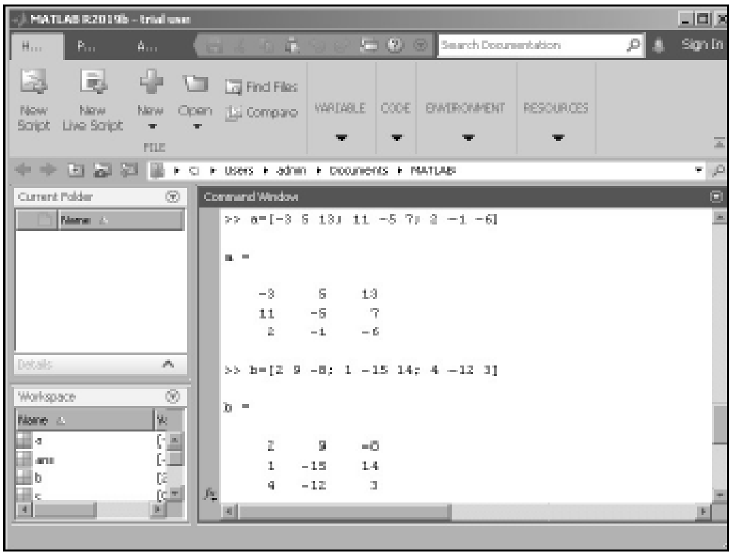

Figure 1. Initial data

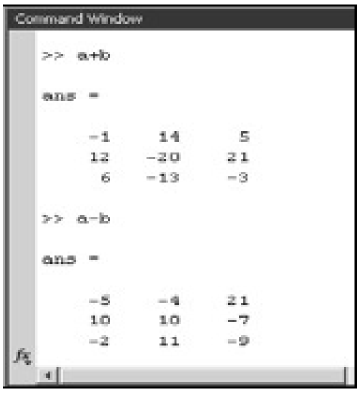

Figure 2. Sum and disparity result

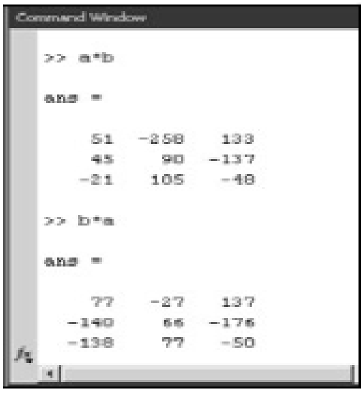

Figure 3. Multiplication result

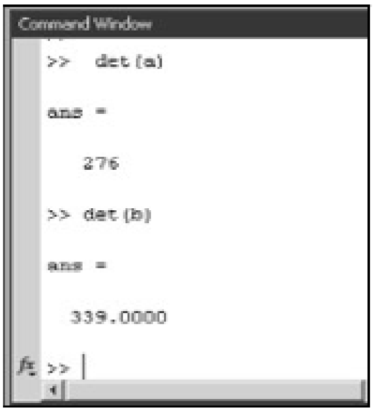

Figure 4. Result of finding the determinant

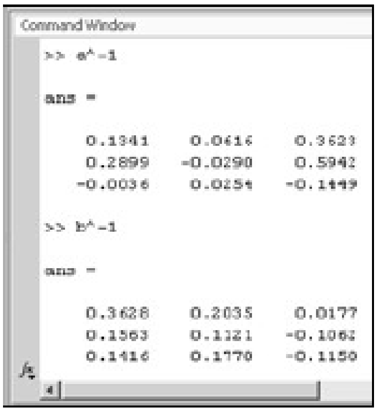

Figure 5. Result of inverse matrix finding

In the example, the matrices are not shown. Operations are executable. Linear equation system. The system of linear equations is given:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = c_1, \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = c_2, \\ \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = c_n. \end{cases}$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} - \text{ the matrix of coefficients;}$$

$$C = \begin{pmatrix} c_1 \\ \cdots \\ c_n \end{pmatrix} - \text{column vector of free members;}$$

$$X = \begin{pmatrix} x_1 \\ \cdots \\ x_n \end{pmatrix} - \text{column vector of unknowns;}$$

Given $|A| \neq 0$, the system has a single solution.

Example:

$$A = \begin{pmatrix} -3 & 5 & 13 \\ 11 & -5 & 7 \\ 2 & -1 & -6 \end{pmatrix}, \quad C = \begin{pmatrix} 46 \\ 22 \\ -18 \end{pmatrix}.$$

Let's consider three ways to solve a system of linear equations using Matlab.

Table 1. *Ways to solve a system of linear equations using Matlab*

1 way. Using the built-in function	
Function linsolve(A,C)	>> X=linsolve(A,C) X = 1.0000 1.0000 1.0000
2 way. Using matrix division right to left	
The matrix determinant A is calculated Vector X.	>>det(A) >>X=A\B X = 1.0000 1.0000 1.0000

3 way. Gauss method	
Construct the extended system of linear algebraic equations matrix using horizontal concatenation.	>>P = [A, C] –3 5 13 46 11 – 5 7 22 2 – 1 – 6 – 18
Reduce the matrix P to a triangular form, using the built-in rref (P) function.	>>P = rref(C) P = 1.0000 0.0000 0.0000 1.0000 0.0000 1.0000 0.0000 1.0000 0.0000 0.0000 1.0000 1.0000
Assign the value of the last column of the matrix P to the vector X.	>>X = P(: , 4) X = 1.0000 1.0000 1.0000

If the course of the solution is not important, then the command "a\c" is used. To solve the system of linear equations by the Cramer method, it is necessary: to set the main matrix of coefficients for unknowns; set auxiliary matrices; calculate unknown systems of equations by dividing the determinant of the corresponding auxiliary matrix by the determinant of the main matrix.

To solve linear systems or nonlinear equations, Matlab has a special solve function. It is necessary: to determine symbolic variables, calculate unknowns by the formula [x1, x2, ...] = solve ('equation1', 'equation2', ...), derive the found solution with a given accuracy using the vpa function (variable, number of characters).

The graphical method can be applied to systems of dimension 2 or 3. It is necessary: to set symbolic variables, a function, design a function graph, add grid lines to the graph, use the graphic "magnifier" to scale the graph to achieve the required accuracy in determining the roots of the equation, the abscissas of the intersection points of the graph of the function are the roots of the equation.

To study the system of linear equations for compatibility and finding the only solution, the script file can be developed:

```
        disp ('System Compatibility Study')
disp ('')% display an empty string on the screen
disp ('Main matrix:')
A = [- 397 399 407; 405-399 401; 396 -395 -400]
disp ('Number of equations:')
n = length (A (:, 1))
disp ('Free member vector:')
C = [440, c. 22, c. -18]
disp ('Extended matrix:')
AC = [A C]
disp ('Extended matrix rank:')
rAB = rank (AC)
disp ('Rank of the main matrix:')
rA = rank (A)
% Compatibility Check
% if r (AC) is not equal to r (A), then the system is incompatible if rAC ~ = rA
        disp ('System is incompatible:')
% if r (AC) = r (A) = n, then the system has a unique solution
elseif (rAC == rA) & (rA == n)
disp ('The system has a single solution:')
X = linsolve (A, C)
disp ('Check:')
C_ = A * X
% if r (AC) = r (A) <n, then the system has infinitely many solutions
else (rAC == rA) & (rA <n)
disp ('The system has infinitely many solutions:')
end
```

There is the development of educational courses with automatic verification of homework, the integration of Matlab with modern and inexpensive equipment, a large number of learning video courses, documentation and many examples of code (Garrison, D and Vaughan, N., 2007). For example, Matlab Grader is a free teaching and learning environment based on Matlab. It designs the interactive learning courses, automatically evaluates students' work in real time, views the results of students completing individual tasks, compiles performance analysis for each student, establishes a weighted method for assessing of test tasks, uses libraries of reusable examples of courses and tasks, adapts courses for use outside the desktop for all users, integrates courses into the learning management system. To develop the interactive training courses, the teacher needs to create an account on www.mathworks.com. It should be linked to the university's license. After the registration, it becomes possible to develop a new course or use the available layouts and adapt them to the requirements. At a new learning course development, it is necessary:

- develop task's description.
- write the supporting decision.
- make a blank for students.
- write tests.
- check your decision.

It is necessary to add students to the course. Students are notified by email and receive a link to access the course. They should log in using their mathworks.com account password and email address. Matlab also has the ability to develop the interactive courses in the Live Editor. This application allows creating lectures, laboratory and practical works combined theoretical material, mathematical equations, program code and code execution results. Each block of the document can be launched independently and visualized code execution immediately after its execution.

274 first-year students of the Faculty of Physics, Mathematics and Informatics of Kherson State University took part in the pedagogical experiment on the introduction of MATLAB in the course «Linear Algebra». During experiment the following tasks were solved: study on formation of mathematical competence;

disclosure of methodical features of teaching the course «Linear Algebra» with the use of ICT; introduction of the MATLAB software environment in the process of learning linear algebra; assessment of the effective use of the environment in the discipline teaching process.

At the theoretical stage, the curricula in the «Linear Algebra» discipline for specialties were analyzed. The possibility of linear algebra elements using in the studying process was examined. The methods for determining the effectiveness of experimental techniques were selected, as well as the forms of experiment conducting and results controls were established.

At this stage there was compiling of theoretical material that must correspond to the content of the curriculum, as well as the selection of problems in linear algebra for practical solutions that can be proposed during practical classes and which can be implemented using the MATLAB environment. During theoretical stage student surveys, individual interviews and pedagogical observation were used.

The formative stage of the experiment is characterized by the introduction of the MATLAB environment in the learning of elements of linear. Homogeneity and representativeness of experimental and control groups participated as respondents, regardless of their progress, interests, abilities etc. The homogeneity of the group was confirmed with the help of Student's t-test. The purpose of the formative experiment was to determine the effectiveness of MATLAB and to confirm or simplify the proposed hypothesis. The main task of the experiment was to evaluate the efficiency of student learning with the implementation of the experimental factor, i.e. the MATLAB environment, into the experimental group and without the introduction of the experimental factor in the control group. The experiment was conducted during study hours. In the experimental group students used the MATLAB environment during studying the elements of linear algebra in the general course «Linear Algebra», and students in the control group studied topics in linear algebra in the traditional mode of learning. Depending on the changes in the amount of hours, the distribution of practical and lecture classes may vary within the workload. Expected results of the experiment: as a result of using the MATLAB environment, the quality of learning the elements of linear algebra and the efficiency of learning increase.

At the final stage of the experiment, the results of the experimental research were processed and generalized, and conclusions were formulated. The impact of experimental learning was determined by the indicator of cognitive (knowledge) and activity (skill). Student's t-test was used to identify the differences in the levels of formation of mathematical competencies in linear algebra between the control and experimental groups. The study under hypothesis H_0 considered the statement that the levels of formation of mathematical competencies in linear algebra in the samples differ slightly. The results testify that the null hypothesis is rejected with a risk of $\alpha=0.05$. Therefore, it can be claimed that with a reliability of 0.95 between the experimental and control groups there are significant differences in the levels of formation of mathematical competencies in linear algebra after the experiment.

The results of the test tasks and the final independent study are shown in diagrams 1,2, where the experimental group results are indicated in black.

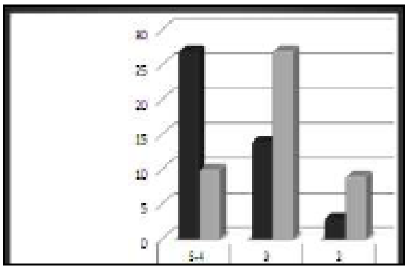

Figure 6. Results of cognitive component formation

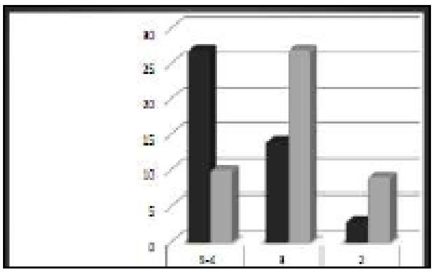

Figure 7. Results of activity component formation

The analysis of the results of the pedagogical experiment revealed and increase in the level of mathematical competencies in linear algebra in students of experimental groups because of the implementation of the MATLAB environment. The experiment showed that the students of the experimental group have a better understanding of the basic concepts of the linear algebra course, typical algorithms for solving problems;

the ability to determine the most appropriate solutions; to model independent learning. It can be stated that the experimental learning of the linear algebra elements provides a higher level of mathematical competence of students.

The effectiveness of information and communication technologies use in teaching Mathematics in universities is achieved at the following conditions:

1. The availability and provision of positive motivation for learning, cognitive activity of students through the use of information and communication technologies.

2. Willingness and ability of students to master new knowledge, skills and abilities in the use of information technology in educational and future professional activities.

3. Organization of students' self-activity using electronic tutorials, including theoretical, practical, test and control material, guidelines for solving computer-oriented problems.

4. Ensuring interactive cooperation in the learning process between students and teachers through information and communication technologies. Using IT for self-organization and self-monitoring of educational activities.

Introducing the capabilities of the Matlab package into the learning process allows improving the students' professional level and forming their interest in learning.

One of the significant advantages of the Matlab system is its integration into almost all areas of modern science and technology. Matlab is a global standard in higher education and research. The linear algebra teaching is based on the traditional presentation of the material. Matlab package application allows to increase the educational process efficiency and to form competencies necessary in future professional activities (Gilat, A., 2004).

Students are open to all kinds of technologies in Mathematics. This openness, together with the availability of high-tech tools, is changing the approach to linear algebra teaching.

Currently, in many educational institutions, the organizing forms of the educational process are being reformed and distance learning is being intensively introduced. It requires the development of electronic textbooks and practical works.

So, the task of increasing the laboratory works efficiency by working only with a Web browser is actual. Tools for remote development have appeared in some mathematical packages, including Matlab. Web application development is a special feature of Matlab to use remote computing for solving tasks efficiency increasing. It greatly facilitates teachers' work and plays an important role in improving the effectiveness of educational organization tools and the distance learning introduction. In the following research, the question of online learning of linear algebra using Matlab will be studied.