Potential accuracy of speckle-interferometric measurements. Binary stars

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The limiting possibilities of speckle-interferometric measurements of binary stars are investigated and the factors limiting them are analyzed. The potential accuracy in estimating the separation, position angle, and brightness difference of the components is investigated for the separate and joint estimation of these parameters. The limiting stellar magnitude for these circumstances of observation is determined in connection with the solution of a concrete measurement problem. The results obtained permit the planning of an experiment on speckle-interferometric measurements of binary stars.

INTRODUCTION

The application of the method of speckle interferometry to the investigation of binary stars has proven most fruitful. The combined efforts of the observers using this method have revealed several dozen new systems having components separations down to 0".02. Regular observations of systems known earlier only as spectral binaries ultimately enabled McAlister to determine the masses and luminosities of the components for nine of them.

Naturally, measurements made with a high accuracy are required to construct high-quality orbits and reliably determine the fundamental astrophysical parameters. The results of measurements of binary stars presented in Ref. 1 well illustrate the accuracy of the method of speckle interferometry. The best measurements have an rms error of less than 1% for the angular distance between the components and 0°.1 for the position angle. In this, the concluding part of the present work, we investigate the potential accuracy of speckle-interferometric measurements of binary stars.

In the second paper of the present work[2] we determined, in general form the covariation matrix of errors of jointly effective (according to Kramer–Rao) estimates of the components of the parametric vector $\mathbf{p} = \mathbf{p}(p_1, p_2, \ldots, p_N)$ of an arbitray object from the results of measuring the power spectrum of speckle images, and we discussed the conditions for obtaining such estimates. In doing this, we took into account the influence of speckle noise, due to atmospheric turbulence, and quantum fluctuations of the light flux[3] on the results of the measurements.

In an individual estimate of the components $p_1, p_2, \ldots, p_N$ (when all of them but one, e.g., p_j, are assumed to be known exactly), the Kramer–Rao boundary condition for the dispersion of the estimate has the form[2,3]

$$\sigma_j^2 = \frac{1}{2n_{0s}^2(M-1)K_j}, \tag{1}$$

where

$$K_j = \int_0^{\pi}\int_0^{\nu_D} \frac{[g_D(\nu)]^2\left[\dfrac{\partial\Phi(\nu,\psi,\mathbf{p})}{\partial p_j}\right]^2 \nu\, d\nu\, d\psi}{[1+n_{0s}g_D(\nu)\Phi(\nu,\psi,\mathbf{p})]^2}, \tag{2}$$

n_{0S} is the average number of detected phonons per speckle, M is the number of speckle images $\Phi(\nu,\psi,\mathbf{p}) = |f_H(\nu,\psi,\mathbf{p})|^2$ is the normalized square of the modulus of the Fourier transform of the brightness distribution over the object, $g_D(\nu)$ is the diffraction optical transmission function (OTF) of the telescope, and $\nu_D = D/\lambda$ is the spatial frequency limited by diffraction in the telescope.

When the components $p_1, p_2, \ldots, p_N$ are estimated jointly, their estimates will be correlated in general. The correlation coefficient for jointly effective estimates of p_j and p_ℓ is determined by the expression[2]

$$g_{jl} = \frac{K_{jl}}{\sqrt{K_j}\sqrt{K_l}}, \tag{3}$$

where

$$K_{jl} = \int_0^{\pi}\int_0^{\nu_D} \frac{[g_D(\nu)]^2\left(\dfrac{\partial\Phi}{\partial p_j}\right)\left(\dfrac{\partial\Phi}{\partial p_l}\right)\nu\, d\nu\, d\psi}{[1+n_{0s}g_D(\nu)\Phi(\nu,\psi,\mathbf{p})]^2}, \tag{4}$$

while K_j and K_ℓ have the same meaning as in (2).

Equations (1)-(4) are valid under the following assumptions: 1) The light receivers are linear in all stages up to the measurement of the power spectra; 2) The exposure time t_E per speckle image and the width $\Delta\lambda$ of the spectral band satisfy the conditions $t_E \leq \tau_S$ (Ref. 4) and $\Delta\lambda/\lambda \lesssim r_0/D$ (Ref. 5), where τ_S is the characteristic "lifetime" of the speckles (1-10 msec), λ is the effective wavelength of the radiation being received, and r_0 is the coherence radius of the wave front disturbed by the atmosphere (Fried's parameter[6]); 3) the size of the object does not go beyond the limits of the isoplanatic region[7]; 4) $(D/r_0)^2 \gg 1$, which is always valid for large telescopes in the visible range, since $r_0 \approx 10$ cm under typical conditions, and $r_0 \lesssim 20$ cm in the general case[8]; 5) the telescopic aberrations are negligibly small for regions of size r_0. We note that the assumptions (2) and (3) are fully natural for speckle interferometry, while their violation can be taken into account by using the results of the work of Roddier et al.[4,7] We also recall that our analysis is confined to the case of $n_T > 1$, of practical interest, where n_T is the average number of photons detected per speckle image.

STATEMENT OF THE PROBLEM

We shall assume that the detection problem has already been solved, and only the problem of measuring (estimating) the unknown parameters remains. The model of the object figuring as the

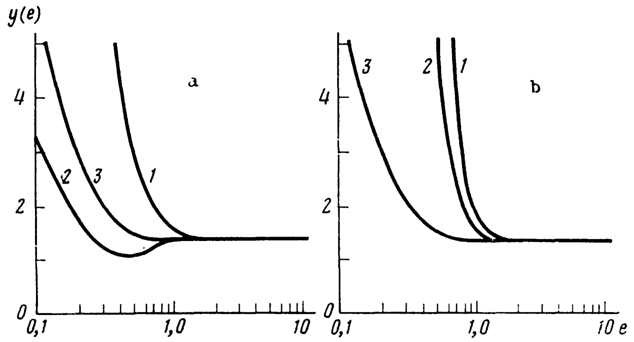

FIG. 1. Root-mean-square errors of estimates of the parameters B, e, and ϕ as functions of $e = \rho D/\lambda$ in the case of $n_{s1}(1+\beta) \ll 1$: a) for separate estimation of the parameters; 1) $y(e) = n_{s1}(1+\beta)\sqrt{M-1}\,\sigma_B$; 2) $y(e) = \beta n_{s1}\sqrt{M-1}\,\sigma_e/0.68(1+\beta)$; 3) $y(e) = \beta n_{s1} e\sqrt{M-1}\,\sigma_\phi/0.68(1+\beta)$. b) Analogous curves for the joint estimation of the parameters, where σ_B^*, σ_e^*, and σ_ϕ^* figure instead of σ_B, σ_e, and σ_ϕ.

a priori information consists of two incoherent point sources. Then the angular distance ρ between the components can be determined from the period of the cosinusoidal function in the power spectrum, the position angle ϕ can be determined from the orientation of the spectrum relative to a given direction, while the brightness difference Δm between the components can be determined from the contrast of the cosinusoid. The typical problem of speckle-interferometric measurements of visual and spectral binaries is set up in just this way. It always arises when the apparent size of each component is negligibly small compared with the distance between the components.

A speckle image of a binary star consists of two identical speckle patterns shifted relative to each other. They can only be distinguished by brightness in accordance with the brightnesses of the components, if, of course, their separation ρ does not go beyond the limits of the isoplanatic region. In this case, the quantity n_{0s} in Eqs. (1), (2), and (4) obviously is

$$n_{0s}=n_{s1}+n_{s2}=n_{s1}(1+\beta), \tag{5}$$

where n_{s1} is the average number of photographic events per speckle from the primary component while

$$\beta=\frac{n_{s2}}{n_{s1}}=10^{-0.4\Delta m}, \tag{6}$$

where Δm is the difference between the brightnesses of the components.

Just as in the case of extended sources, to which the first two papers[2,3] of the present work were devoted, we introduce a measure of the closeness of the components of the binary (e), and we define it as the ratio of the separation ρ to the diffraction element of resolution (λ/D):

$$e=\frac{\rho D}{\lambda}. \tag{7}$$

Then the normalized square of the absolute value of the object's spectrum, in polar coordinates ξ, ψ, where $\xi = \nu/\nu_D$ $(0 \le \xi \le 1)$, is described by the expression

$$\Phi(\xi,\psi)=1+B\{\cos[2\pi e\xi\cos(\psi-\varphi)]-1\}, \tag{8}$$

where

$$B=\frac{2\beta}{(1+\beta)^2}. \tag{9}$$

The quantity B equals 0.5 for $\Delta m = 0$ and it approaches zero as $\Delta m \to \infty$.

First we investigate the potential accuracy of the estimates of Δm, ρ, and ϕ when the parameters are estimated separately, and then we proceed to the case, very important in practice of the joint estimation of the parameters.

ACCURACY IN ESTIMATING THE BRIGHTNESS DIFFERENCE OF THE COMPONENTS

First we find the potential accuracy in estimating the parameter B. In solving this problem for an arbitrary level of detected signal, we must resort to numerical integration over both coordinates. At low light levels, however, i.e., for $n_{s1}(1+\beta) \ll 1$, the problem can be reduced to the calculation of a single integral. In this case, the term $n_{0s}g_D(\xi)\Phi(\xi,\psi,p)$ in the denominator of the integrand in (2) can be neglected. Thus, we arrive at the integral

$$K_B=\frac{4}{\pi^2}\int_0^\pi\int_0^1\{\cos[2\pi e\xi\cos(\psi-\varphi)]-1\}^2(\arccos\xi-\xi\sqrt{1-\xi^2})^2\,\xi\,d\xi\,d\psi. \tag{10}$$

Here, as everywhere below, the OTF of a telescope with a round continuous aperture is used as $g_D(\xi)$. Removing the curly brackets and using the relation $\cos^2 x = (1+\cos 2x)/2$, as well as the well-known integral[9]

$$\int_0^\pi \cos(a\cos x)\,dx=\pi J_0(a), \tag{11}$$

where J_0 is a zeroth-order Bessel function, after integrating over ψ, we obtain

$$K_B=\frac{2}{\pi}\int_0^1[3+J_0(4\pi e\xi)-4J_0(2\pi e\xi)](\arccos\xi-\xi\sqrt{1-\xi^2})^2\,\xi\,d\xi. \tag{12}$$

Substituting (12) into (1) and designating $K_B = 2H_B/\pi$, for quantity σ_B we find

$$\sigma_B=\frac{\sqrt{\pi}}{2n_{s1}(1+\beta)\sqrt{(M-1)H_B}},\quad n_{s1}(1+\beta)\ll 1. \tag{13}$$

The integral H_B (like other single integrals that

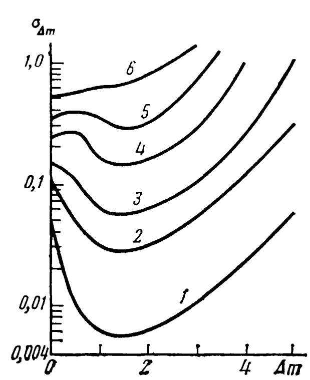

FIG. 2. Dependence of the rms error $\sigma_{\Delta m}$ in estimating the brightness difference Δm: 1) for σ_B = 0.001; 2) 0.005; 3) 0.01; 4) 0.025; 5) 0.05; 6) 0.1.

we shall encounter below) was calculated for different e using a Nairi-K computer. The graph of σ_B as a function of e, normalized in such a way that it does not depend on the concrete conditions (i.e., on Δm, n_{s1}, or M), is presented in Fig. 1a (curve 1). The following expression is valid for $e \gg 1$:

$$\sigma_B = \frac{1.36}{n_{s1}(1+\beta)\sqrt{M-1}}, \quad e \gg 1,\ n_{s1}(1+\beta) \ll 1. \tag{14}$$

Now we must convert from σ_B to $\sigma_{\Delta m}$. This is easy to do for large Δm. In fact, for $\beta \ll 1$ we get, from (9),

$$B \simeq 2\beta = 2 \cdot 10^{-0.4\Delta m}. \tag{15}$$

Hence,

$$\Delta m = 2.5(\lg 2 - \lg B). \tag{16}$$

And we estimate the quantity $\sigma_{\Delta m}$ in the well-known way:

$$\sigma_{\Delta m} = \left|\frac{d(\Delta m)}{dB}\right| \sigma_B = \frac{1{,}085}{B}\sigma_B = 0.54 \cdot 10^{0.4\Delta m}\sigma_B. \tag{17}$$

To find the exact transition from σ_B to $\sigma_{\Delta m}$ for all values of Δm, one must know the probability density function of the estimate $\hat{B}$. Let the true value of B in the power spectrum being measured be B_0. By virtue of the central limit theorem, the quantity $\hat{B} - B_0$ is has an asymptotically normal distribution with a zero mean and a standard deviation σ_B.

Further, by substituting (6) into (9) and solving the equation for Δm, it is easy to ascertain that a solution (out of two possible ones) of it will be

$$\Delta m = 2.5[\lg B - \lg(1 - B + \sqrt{1-2B})]. \tag{18}$$

Then

$$\sigma_{\Delta m}^2 = \frac{6.25}{\sigma_B\sqrt{2\pi}} \int_{0,5}^{0} [\lg B - \lg(1-B+\sqrt{1-2B})]^2 \times \exp\left[-\frac{(B-B_0)^2}{2\sigma_B^2}\right] dB - \left\{\frac{2.5}{\sigma_B\sqrt{2\pi}} \int_{0,5}^{0} [\lg B - \lg(1-B+\sqrt{1-2B})] \times \exp\left[-\frac{(B-B_0)^2}{2\sigma_B^2}\right] dB\right\}^2. \tag{19}$$

Calculations through Eq. (19) were made for values of Δm and σ_B satisfying the condition $\sigma_B \lesssim B$. The dependence of $\sigma_{\Delta m}$ on Δm for different values of σ_B is shown in Fig. 2.1 It is seen from Fig. 2 that for low values of σ_B, such that the approximation $\sigma_{\Delta m} \approx \left|\frac{d(\Delta m)}{dB}\right| \sigma_B$, is valid, which follows from the possibility of representing the nonlinear function $\Delta m = \Delta m(B)$ by a Taylor series with allowance for only first-order terms, the function $\sigma_{\Delta m} = \sigma_{\Delta m}(\Delta m)$ has a clear minimum. The presence of this minimum means that it is just as hard to estimate values of Δm close to zero with a high accuracy as it is for larger Δm.

From (13) and (14) it follows that the quantity σ_B depends little on Δm and approaches a finite limit as $\Delta m \to \infty$. From Fig. 2, as well as form (17), it is seen that the quantity $\sigma_{\Delta m}$ grows without limit as $\Delta m \to \infty$.

ACCURACY OF SEPARATE ESTIMATES OF THE PARAMETERS ρ AND ϕ

Let us dwell in more detail on the case of $n_{s1} \cdot (1+\beta) \ll 1$. Substituting the expressions for $\partial\Phi/\partial e$ and $\partial\Phi/\partial\phi$ into (2) successively, and neglecting the term $n_{0s}g_D(\xi)\Phi(\xi, \psi, p)$ in the denominator, as we did before, we integrate over ψ, for which we first reduce the degree of the trigonometric functions, expressing them through the cosines of twice the angles, and then we use the well-known integral[9]

$$\int_0^\pi \cos(a\cos x)\cos nx\,dx = \pi\cos\left(\frac{n\pi}{2}\right)J_n(a), \tag{20}$$

where J_n is an n-th-order Bessel function.

As a result, for K_e and K_ϕ we obtain

$$K_e = 4\pi B^2 \int_0^1 [1 - J_0(4\pi e\xi) + J_2(4\pi e\xi)](\arccos\xi - \xi\sqrt{1-\xi^2})^2\xi^3\,d\xi, \tag{21}$$

$$K_\varphi = 4\pi B^2 e^2 \int_0^1 [1 - J_0(4\pi e\xi) - J_2(4\pi e\xi)](\arccos\xi - \xi\sqrt{1-\xi^2})^2\xi^3\,d\xi = 4\pi B^2 e^2 \int_0^1 \left[1 - \frac{J_1(4\pi e\xi)}{2\pi e\xi}\right](\arccos\xi - \xi\sqrt{1-\xi^2})^2\xi^3\,d\xi, \tag{22}$$

where J_0, J_1, and J_2 are Bessel functions of zeroth, first, and second orders, respectively.

Designating what is inside the integral in (21) as H_e and in (22) as H_ϕ for σ_e and σ_ϕ we finally find

$$\sigma_e = \frac{1}{2Bn_{s1}(1+\beta)\sqrt{2(M-1)\pi H_e}}, \tag{23}$$

$$\sigma_\varphi = \frac{1}{2Bn_{s1}(1+\beta)e\sqrt{2(M-1)\pi H_\varphi}}, \quad n_{s1}(1+\beta) \ll 1. \tag{24}$$

Graphs of the dependence of σ_e and σ_ϕ on e, normalized so as not to depend on σm, n_{s1}, or M, are presented in Fig. 1a (curves 2 and 3). For $e \gtrsim 1$, the integrals H_e and H_ϕ cease to depend on e, and in this case the following expressions for σ_e and σ_ϕ are valid:

$$\sigma_e = \frac{1{,}36}{Bn_{s1}(1+\beta)\sqrt{M-1}} = \frac{0{,}68(1+\beta)}{\beta n_{s1}\sqrt{M-1}}, \tag{25}$$

$$\sigma_\varphi = \frac{\sigma_e}{e}, \quad e \gg 1,\ n_{s1}(1+\beta) \ll 1. \tag{26}$$

The case of high detected signal levels is presented in Fig. 3. In it we give graphs of the dependence of the rms error σ_e in estimating the measure of closeness, multiplied by $\sqrt{M-1}$, on its value e, when $n_{s1} = 1$ and 10^2 photographic events per speckle, and $\Delta m = 0$ and 5^m. The calculations required for this were made on an ES-1052 computer (all the double integrals in this paper were taken with this computer). From Fig. 3 it is seen that, first, for $e > 1$ the rms error in estimating the measure of closeness of the components of binary stars does not vary with an increase in e, remaining constant for the given Δm, n_{s1}, and M. Second, 100-fold variation of the total signal level (in the case of $n_{s1} \gtrsim 1$) had considerably less influence on σ_e than the same variation in the brightness of the satellite relative to the brightness of the primary component.

JOINT ESTIMATION OF THE PARAMETERS

First we consdier the case of $n_{s1}(1+\beta) \ll 1$. To find the correlation coefficients of the jointly effective estimates B, e, and ϕ, we substitute the ex-

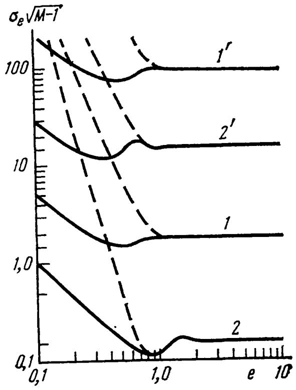

FIG. 3. Dependence of the rms error σ_e in estimating the measure of closeness of the components of a binary star, multiplied by $\sqrt{M-1}$, on its value $e = \rho D/\lambda$: 1) $n_{s1} = 1$, $\Delta m = 0$; 1') $n_{s1} = 1$, $\Delta m = 5^m$; 2 and 2') $n_{s1} = 10^2$ for $\Delta m = 0$ and 5^m, respectively. Dashed lines correspond to the case of a joint estimate of the parameters.

pressions for $\partial\Phi/\partial B$, $\partial\Phi/\partial e$ and $\partial\Phi/\partial\phi$ into (4) in pairs and integrate over ψ. Since[9]

$$\int_0^\pi \sin(a\cos x)\sin nx\,dx=0,$$
$$\int_0^\pi \sin(a\cos x)\cos nx\,dx=\pi\sin\frac{n\pi}{2}J_n(a), \tag{27}$$

we find that the integrals $K_{e\phi}$ and $K_{B\phi}$ vanish, and hence $q_{e\phi} = 0$ and $q_{B\phi} = 0$. The correlation coefficient of the estimates of the parameters B and e proves to be

$$q_{Be}=-\sqrt{2}\frac{H_{Be}}{\sqrt{H_B}\sqrt{H_e}}, \quad n_{s1}(1+\beta)\ll 1, \tag{28}$$

where

$$H_{Be}=\int_0^1 [2J_1(2\pi e\xi)-J_1(4\pi e\xi)](\arccos\xi-\xi\sqrt{1-\xi^2})^2\xi^2\,d\xi, \tag{29}$$

while H_B and H_e have the same values as in (13) and (23), respectively.

Graphs of q_{Be} as a function of e for $n_{s1} = 10^2$ and 1 and $n_{s1}(1+\beta) \ll 1$ are shown in Fig. 4. It is seen from this figure that, first, the correlation coefficient decreases in absolute value with an increase in n_{s1}. From this, as well as from the equality of $q_{e\phi}$ and $q_{B\phi}$ to zero for $n_{s1}(1+\beta) \ll 1$, it follows that $q_{e\phi} = 0$ and $q_{B\phi} = 0$ for an arbitrary level fo the detected signal. Second, the value of q_{Be} depends little on the difference between the brightnesses of the components. Thus, for $n_{s1} = 10^2$ photographic events per speckle, the values of q_{Be} for $\Delta m = 0$ and $\Delta m = 5^m$ (curves 1 and 1') differ little from each other, while for $n_{s1} = 1$ there is no difference at all.

The presence of correlation between estimates of the parameters B and e for $e \lesssim 1$ means that, except for the case of $n_{s1} \gtrsim 1$, it is hard to distinguish their actions on the shape (profile) of the power spectrum, while for $e \lesssim 0.3$ it is practically impossible to do this in any case. It should be noted that in this region of values of e, where q_{Be} differs from zero, the effects of the influence of B and e on the shape of the spectrum are of the same sign, while the negativity of q_{Be} compensates for these influences.

Consequently, if a priori information on the difference in the brightnesses of the components is absent, there is practically no possibility of measuring the angular separation for $e \lesssim 0.3$, or for $e < 0.5$ for low signal light levels ($n_{s1} < 1$).

Knowing q_{Be}, $q_{B\phi}$, and $q_{e\phi}$ and the rms errors σ_B, σ_e, and σ_ϕ of the separately effective estimates one can find the rms errors σ_B^*, σ_e^*, and σ_ϕ^* of the jointly effective estimates of the unknown parameters. We have[10]

$$\sigma_B^{*2}=\frac{\sigma_B^2}{1-q_{Be}^2},\quad \sigma_e^{*2}=\frac{\sigma_e^2}{1-q_{Be}^2},\quad \sigma_\phi^{*2}=\sigma_\phi^2. \tag{30}$$

As follows from the behavior of q_{Be}, the equalities $\sigma_B^* = \sigma_B$ and $\sigma_e^* = \sigma_e$ are valid for $e > 1.5$, when $n_{s1} \lesssim 1$, and for $e > 1$ in all other cases. Serving as illustrations of this are Fig. 1b, in which graphs of the dependence of σ_B^*, σ_e^*, and σ_ϕ^* on e for the case of $n_{s1}(1+\beta) \ll 1$ are shown, and Fig. 3, in which the dependence of σ_e^* on e in the case of high light levels is represented by dashed lines. Thus, in a joint estimate of the parameters B, E, and ϕ, when $e > 1$ (when $e > 1.5$ for $n_{s1} \lesssim 1$), the potential accuracy in estimating each of the parameters proves to be the same as in the case when it is estimated while the other two parameters are known exactly.

Thus, the accuracy in solving the problem of measuring a binary star was also investigated for the case when a priori information about the object is reduced to a minimum and consists only in the fact that it is a binary star described by a model of two point sources.

LIMITING STELLAR MAGNITUDE

Let us estimate what the stellar magnitude of the object must be if, under given observing conditions, one must obtain 1% accuracy in a separate estimate of e when e = 1. This is easy to do when $n_{s1}(1+\beta) \ll 1$, which means $n_{s1} \ll 0.5$ for $\Delta m = 0$. Let $M = 10^6$. From (25) we find the required number of photographic events per speckle (on the average): $n_{s1} \approx 0.14$. On the other hand, $n_{s1} = n_{T1}/N_{s1}$, where n_{T1} is the average number of detectable photons per speckle image of the primary component, while N_{s1} is the average number of speckles in this image. The quantity N_{s1} (it is

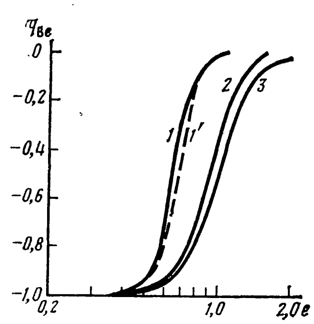

FIG. 4. Correlation coefficient of jointly effective estimates of the parameters B and e as a function of $e = \rho D/\lambda$: 1) $n_{s1} = 10^2$, $\Delta m = 0$; 1') $n_{s1} = 10^2$, $\Delta m = 5^m$; 2) $n_{s1} = 1$, $\Delta m = 0\text{-}5^m$; 3) case of $n_{s1}(1+\beta) \ll 1$.

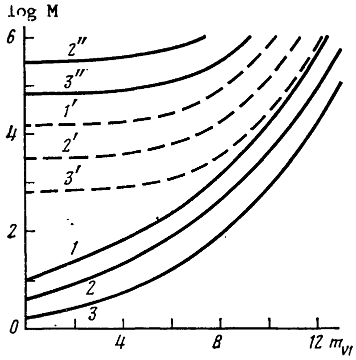

FIG. 5. Number M of speckle images required to achieve an accuracy of 1% of the stellar magnitude m_{V1} of the primary component when estimating the parameter $e = \rho D/\lambda$: 1 and 1′) e = 1 for Δm = 0 and $2^m.5$, respectively; 2, 2′, and 2″) e = 5 for Δm = 0, $2^m.5$, and 5^m; 3, 3′, and 3″) the same Δm, but e = 25.

obvious that $N_{S1} = N_{S2}$ always) is usually taken as[11, 12]

$$N_{s1} = \frac{1}{0.435}\left(\frac{D}{r_0}\right)^2 . \qquad (31)$$

Knowing N_{S1}, it is easy to show that

$$n_{s1} = 0.342\eta n_{V1} t_E \Delta\lambda r_0^2, \qquad (32)$$

where n_{V1} is the spectral density of the flux from the primary component (quanta/cm²·sec·nm), while by the quantity η we understand, as in the first two papers of the present work, the equivalent quantum yield of the speckle camera.[13, 14] We assume that the obseving conditions are these: r_0 = 10 cm, t_E = 0.01 sec, $\Delta\lambda$ = 40 nm, and η = 10%. Then the spectral flux density of the primary component (from each of the components for $\Delta m = 0$) must be n_{V1} = 0.1 quanta/cm²·sec·nm. Since the flux density of a zero-magnitude star in the green region of the spectrum is[15] $n_0 = 10^4$ quanta/cm²·sec·nm, we have

$$m_{V1} = 2.5 \lg \frac{n_0}{n_{V1}} = 12^m.5.$$

In a joint estimate of the parameters, we have $m_{V1} \approx 12^m.3$. The limiting stellar magnitude required to achieve the same accuracy, but for e = 10, will be $m_{V1} = 15^m$ in both cases. If absolute accuracy must be retained with an increase in e, the m_{V1} does not increase.

In the general case, we can calculate the sample volume M required to solve a given measurement problem as a function of the brightness of the object. In Fig. 5 we show the dependence of M on m_{V1}, corresponding to the demand σ_e/e = 1%, when e = 1, 5, and 25 and for Δm = 0, $2^m.5$, and 5^m. This figure enables one to trace the influence of three factors on the size of the sample volume: the brightness difference Δm, and the separation $e = \rho D/\lambda$ of the components (for $e \geq 1$). It is seen from the figure that for bright objects, the brightness difference is the dominant factor. With an increase in m_{V1}, the influence of the brightness of the object gradually comes to dominate. Thus, Fig. 5 provides an idea of the limiting stellar magnitude of objects accessible to the method of speckle interferometry when solving a specific measurement problem.

CONCLUSION

Thus, we have investigated the potential accuracy in estimating unknown parameters in the problem of measuring binary stars by the method of speckle interferometry. The potential accuracy of speckle-interferometric measurements is determined by the type of object and its brightness, the OTF of the telescope, and by the properties of the light receiver, the quantity of speckle images recorded, and the degree of atmospheric turbulence. An important distinguishing feature of these measurements is the dependence of the accuracy on the number M of speckle images recorded and, in the case of low light levels, on the average number of photographic events per speckle to the first power. The dependence $\sigma_p \propto 1/\sqrt{M-1}$ is an important limiting factor for highly accurate measurements of especially faint objects, since in practice it is scarcely possible to acquire and analyze more than 10^6 statistically independent speckle images. Another significant limiting factor is the degree of atmospheric turbulence or the image quality, characterized by Fried's parameter r_0 since in the case of $n_{S1}(1+\beta) \ll 1$, we have $\sigma_p \propto 1/r_0^2$, under the condition that the exposure time (per frame) and the filter band width are properly chosen. The quadratic (at least, as was shown in the first paper[3] of the present work) dependence of the accuracy on r_0 means that as the image quality worsens, speckle-interferometric measurements rapid lose their effectiveness.

Many estimates of the limiting stellar magnitude for the method of speckle interferometry exist (e.g., Refs. 12 and 16-19). But they are all based on rather arbitrarily chosen criteria. A certain value (chosen arbitrarily) of the signal-to-noise ratio, enabling one, as it is stated, either to detect the given object or to measure it, is chosen as some criterion, as a rule. As for the possibility of making a measurement, without a stipulation of the accuracy expected in this case, this concept remains diffuse.

Dainty,[12, 16, 17] e.g., starts from the assumption that measurements of a binary star, both through the autocorrelation function and through the power spectrum, become possible if the signal-to-noise ratio resulting from M images $(S/N)_M$ equals five at a certain point The autocorrelation function for a binary star contains two maxima arranged symmetrically about the origin of coordinates and at a distance of 2e from each other. In this case, the signal has been taken[12, 17] as the heights of these maxima above the background surrounding them, while the noise is the rms fluctuation of this background. Then, for Δm = 0, M = 10^5, D = 4 m, t_E = 0.02 sec, $\Delta\lambda$ = 25 nm, r_0 = 20 cm, and η = 10%, the condition $(S/N)_M$ = 5 leads to the limiting magnitude[12] $m_V \approx 18^m$. In measurements of a power spectrum, besides the condition $(S/N)_M$ = 5 at a arbitrarily chosen point ξ, one must assign the signal $g_D(\xi)|f_n(\xi)|^2$ at this point. This obviously introduces still more uncertainty into the estimate of m_V. For $g_D(\xi)|f_n(\xi)|^2 = 0.2$ (Ref. 12) and the same observing conditions, we find that $m_V = 14^m.7$ (in Ref. 12, $m_V = 13^m.2$, but for r_0 = 10 cm). In this case, the limiting stellar magnitude turns out not to depend on the telescope diameter.

In the present work, the limiting stellar magnitude is determined in connection with the solution of a specific measurement problem. In such a determination, the magnitude m_V proves to depend on the telescope diameter and it increases with an increase in D.

This approach, apart from the solution of the problem of a priori analysis and planning of the experiment, enables one to determine the region of the most effective application of the method of speckle interferometry for measurements of binary stars having a measure of closeness of the components $e > 25$ ($\rho \gtrsim 0''.5$). Having in mind highly exact measurements (to within 1% in separation, let us say) and taking $M \approx 100$ and $\eta = 10\%$, for typical observing conditions we obtain ($r_0 = 10$ cm) $m_{V_1} < 8^m$ for $\Delta m = 0$. Allowing for the inevitable light losses in the atmosphere, the telescope optics, and the speckle camera, we obtain $m_{V_1} \gtrsim 6^m$, and this under the condition that all the information contained in the speckle images is fully used in the measurements. Taking into account the latter condition and the fact that the power spectrum and the autocorrelation function comprise a Fourier-transform pair, we come to the conclusion that these estimates are also valid for digital speckle interferometry,[20] based on the measurement and analysis of autocorrelation functions of speckle images.

For $\Delta m = 2^m.5$ and the same observing conditions, it is already impossible to estimate the separation to within 1%. If $M = 10^5$, then $m_{V_1} < 13^m$ for $\Delta m = 0$, while $m_{V_1} \lesssim 11^m$ with allowance for light losses.

It should be mentioned that the difference in the brightnesses of the components is rarely determined in practice. There are several reasons for this. In mass observations of binary stars, the measurement process is simplified, as a rule, reducing it to purely geometrical measurements with a two-coordinate microscope. As a result, only the parameters ρ and ϕ are determined. Moreover, measurements of band contrast in the power spectrum, yielding information about Δm, can be distorted by effects produced by the reception of a finite band width $\Delta\lambda$ and the finite exposure time t_E, as well as by effects of nonlinearity and partial nonisoplanaticity (for $e \gg 1$). It is difficult to allow for these effects, and this also frequently results in rejecting the determination of Δm.

The present paper completes our investigation of the potentialities of the method of speckle interferometry in the solution of various measurement problems. The author thanks V. N. Dudinov, V. G. Vakulik, and V. S. Tsvetkova for useful discussions.

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