Solution of the problem of optimizing route with using the risk criterion

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Abstract. The aim of the work is to determine the conditions of optimality in the task of plotting the course of the vessel and the operation of divergence of vessels in conditions of intensive navigation. The need for such work is dictated, firstly, by an increase in the intensity of shipping and, secondly, by the emergence of autonomous ships and transport systems, the traffic control algorithms of which obviously require an optimal approach. The criterion of optimality in problems of this class is the expected risk, one of the components of which is the risk of collision of ships. Based on the analysis of methods for constructing ship divergence algorithms, the task is to find a control algorithm that delivers the best results for all participants in the operation. This formulation of the task greatly facilitates the forecast of the actions of all participants in the discrepancy and is especially expedient in the case of participation in the operation of an autonomous system or a ship with which no contact has been established. Theoretically, the task belongs to the most difficult class of control problems - optimal control of a distributed dynamic system with a vector - a goal functional [3, 5, 8, 13–15]. The ability to obtain a general solution to the task of optimal ship control makes this study expedient.

Keywords: vector-functional, optimal control, risk criterion, safe separation, mathematical model, avoid collision

1 Introduction

Research into the problem of effective methods for preventing collisions of ships has become paramount and important in connection with the increase in tonnage, overall dimensions, speed and number of ships involved in the carriage of goods by sea. Particularly important is the factor of the emergence of autonomous ships and systems, the actions of which have a clear algorithm and a specific goal. Thus, it becomes possible to reduce the uncertainty in the task of forecasting the actions of ships, which expands the possible range of actions of own ship. Taking into account the achievements in improving the safety of navigation, the use of radars, and then the development of the ARPA (Automatic Radar Plotting Aids) collision avoidance system, which allows you to automatically track at least 20 encountered objects, determine the parameters of their movement (speed V_n , course Ψ_n), approach elements (($D^n_{min} = DCPA_n$ - distance to the closest point of approach (Distance of the Closest Point of Approach), $T^n_{min} = TCPA_n$ - time to the closest point of approach, (Time to the Closest Point of Approach)), as well as an assessment of the collision risk r_n, the task is to determine the general optimality criteria and methods for solving the task.

The works [18,19,21–24] considered methods of increasing the accuracy and reliability of vessel control in automated and automatic control systems, including avoid collision. A more efficient method is to determine a safe trajectory of the vessel, taking into account the trajectories of all vessels involved in the operation. However, there is considerable uncertainty associated with the actions of the courts in the divergence process. Uncertainty reduction can be achieved if the actions of own ship are consistent with those of other ships. This task is simple but requires the use of optimal control methods under the condition of optimization of the vector - functional. This is already a difficult task, the methods of solving which are poorly studied, but its solution, in this case, allows you to build optimal algorithms for plotting a course when the ships diverge.

The purpose of this article is to substantiate, develop and simulate an algorithm for safe separation of ships using the criterion of expected risk.

2 Problem Statements

The task of optimal plotting of the course first of all requires the determination of the criterion of optimality or the function of the goal. It becomes necessary to plot the trajectory of the vessel $S(x)$ in such a way as to avoid possible collisions, loss of cargo and other complications. This need is formulated as minimization of the risk C on the trajectory of the vessel. Obstacles to navigation are expressed by constraints such as the inequalities $\varphi_i(\mathbf{x}) = 0, i = \overline{1..m}$ and inequalities $\varphi_i(\mathbf{x}) < 0, i = \overline{m..n}$, that is, we obtain the Lagrange task [2,9,20]

$$\mathbf{x}^* \to minC(S(\mathbf{x})),$$

$$\varphi_i(\mathbf{x}) = 0, \overline{1..m}, \tag{1}$$

$$\varphi_i(\mathbf{x}) < 0, i = \overline{m..n}.$$

3 Materials and Methods

The well-known technique for solving this problem involves the formation of the Lagrange function $L(x, \lambda)$, the gradient of which on x^* vanishes

$$\frac{\partial L}{dx} = 0,$$

$$L(x, \lambda) = \lambda_0 S(x) - \lambda_1 \varphi_1(x) - \lambda_2 \varphi_2(x), \quad gradL = 0, \rightarrow \frac{\partial L}{\partial \lambda_1} = 0, \tag{2}$$

$$\lambda_2 \varphi_2(x) = 0.$$

Condition (2), known as the Kuhnna-Tucker theorem [6], defines the optimum point as a point stationary in the coordinate when constraints such as equality are satisfied and the goal function is insensitive to constraints such as inequality. In this simple but important problem, let us trace the meaning of the Lagrange multipliers λ

$$gradL(x, \lambda) = 0 \rightarrow \frac{\partial S}{\partial x} = \lambda \frac{\partial \phi}{\partial x} \rightarrow \lambda \frac{\partial S}{\partial \phi}. \tag{3}$$

Thus, expression (3) illustrates an important fact - the Lagrange multiplier is the sensitivity of the goal function to constraints. Condition (3) plays an important role in the problem of constructing an optimal route. The considered divergence problem differs from the standard one in that it seeks the optimal solution to the divergence problem for all ships. Thus, for n participants in the operation, there are n goal functions and a route laid in such a way that all goal functions reach their optimum

$$\left.\begin{array}{c} x^* \rightarrow minS_1(x) \\ x^* \rightarrow minS_2(x) \\ \vdots \\ x^* \rightarrow minS_n(x) \end{array}\right\} \rightarrow \mathbf{s}(x) = \begin{bmatrix} S_1(x) \\ S_2(x) \\ \vdots \\ S_n(x) \end{bmatrix}. \tag{4}$$

For the simplest optimization problem, the vector of function s is set to the problem $x^* \rightarrow min(s)$, then

$$\left.\begin{array}{c} x^* \rightarrow minS_1(x) \\ x^* \rightarrow minS_2(x) \\ \vdots \\ x^* \rightarrow minS_n(x) \end{array}\right\} \rightarrow \left.\begin{array}{c} \frac{dS_1(x)}{dx} = 0 \\ \frac{dS_2(x)}{dx} = 0 \\ \vdots \\ \frac{dS_n(x)}{dx} = 0 \end{array}\right\}. \tag{5}$$

Condition (5) is satisfied only if the extremum point for all components of the goal function vector coincides; therefore, in the neighborhood of the point x^*, the components of the goal function vector are indistinguishable and the problem is degenerate. As a consequence, there are restrictions on the values of the components of the target vector at the extremum point

$$\left.\begin{array}{c} x^* \to minS_1(x) \\ x^* \to minS_2(x) \\ \vdots \\ x^* \to minS_n(x) \end{array}\right\}; \qquad \left.\begin{array}{c} S_1(x) - a_1 = 0 \\ S_2(x) - a_2 = 0 \\ \vdots \\ S_n(x) - a_n = 0 \end{array}\right\}. \tag{6}$$

Then we obtain the optimality condition (3) in the form

$$\frac{d\mathbf{s}}{dx} = \begin{pmatrix} \lambda_{11} & \dots & \lambda_{1n} \\ \cdot & \cdot & \cdot \\ \lambda_{n1} & \dots & \lambda_{nn} \end{pmatrix} \frac{d(\mathbf{s} - \mathbf{a})}{dx}. \tag{7}$$

Solution of the problem - the matrix of Lagrange multipliers must be unit. Indeed, equation (7) has a solution only for the unit matrix Λ

$$\mathbf{a} = const \to \frac{d(\mathbf{s} - \mathbf{a})}{dx} = \frac{d\mathbf{s}}{dx} \to \frac{d\mathbf{s}}{dx} = \begin{pmatrix} 1 & \dots & 0 \\ \cdot & \cdot & \cdot \\ 0 & \dots & 1 \end{pmatrix} \frac{d\mathbf{s}}{dx}. \tag{8}$$

Taking into account the meaning of Lagrange multipliers (3), we can write down the optimality condition in problem (4)

$$\frac{\partial S_i}{\partial S_j} = 0; \quad i = \overline{1, n}; \quad j = \overline{1, n}; \quad i \neq j; \quad i \neq j \to \frac{\partial S_i}{\partial S_i} = 1. \tag{9}$$

From condition (9) it follows that the optimal solution should not worsen any of the solutions, that is, the components of the goal vector are independent and their states do not affect each other. This result is known as the Pareto criterion or as the effective Jeffrion solution [7].

If the components of the target vector depend on several variables, the problem becomes more cumbersome, since in this case the derivative of the component of the target vector with respect to the state vector turns into a matrix and, as a consequence, the matrix of Lagrange multipliers becomes cellular. Thus, in the problem with the dimension of the target vector equal to the dimension of the state space, we have

$$\left.\begin{array}{c} \mathbf{x}^* \to minS_1(\mathbf{x}) \\ \mathbf{x}^* \to minS_2(\mathbf{x}) \\ \vdots \\ \mathbf{x}^* \to minS_n(\mathbf{x}) \end{array}\right\}; \qquad \left.\begin{array}{c} S_1(\mathbf{x}) - a_1 = 0 \\ S_2(\mathbf{x}) - a_2 = 0 \\ \vdots \\ S_n(\mathbf{x}) - a_n = 0 \end{array}\right\}. \tag{10}$$

Optimality condition (3) in the form (10) takes a more complex form with the cellular matrix of factors

$$\frac{ds}{dx} = \begin{pmatrix} \begin{pmatrix} \lambda_{11}^{11} \cdot \lambda_{1n}^{11} \\ \cdot \quad \cdot \quad \cdot \\ \lambda_{n1}^{11} \cdot \lambda_{nn}^{11} \end{pmatrix} \cdots \begin{pmatrix} \lambda_{11}^{1n} \cdot \lambda_{1n}^{1n} \\ \cdot \quad \cdot \quad \cdot \\ \lambda_{n1}^{1n} \cdot \lambda_{nn}^{1n} \end{pmatrix} \\ \cdot \qquad \cdot \qquad \cdot \\ \begin{pmatrix} \lambda_{11}^{n1} \cdot \lambda_{1n}^{n1} \\ \cdot \quad \cdot \quad \cdot \\ \lambda_{n1}^{n1} \cdot \lambda_{nn}^{n1} \end{pmatrix} \cdots \begin{pmatrix} \lambda_{11}^{nn} \cdot \lambda_{1n}^{nn} \\ \cdot \quad \cdot \quad \cdot \\ \lambda_{n1}^{nn} \cdot \lambda_{nn}^{nn} \end{pmatrix} \end{pmatrix}. \tag{11}$$

Thus, the equation (7) has a solution only for the unit matrix Λ

$$\mathbf{a} = const \to \frac{d(\mathbf{s}-\mathbf{a})}{d\mathbf{x}} = \frac{d\mathbf{s}}{d\mathbf{x}} \to \frac{d\mathbf{s}}{d\mathbf{x}} \begin{pmatrix} \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} \cdots \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} \\ \cdot \qquad \cdot \qquad \cdot \\ \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} \cdots \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} \end{pmatrix}. \tag{12}$$

With the inequality of the dimensions of the target vector and the state vector, we are dealing with non-square matrices, but at the same time the optimality requirements remain - mutual insensitivity at the optimum point between the components of the target vector.

The problem of constructing an optimal trajectory implies optimality along the entire trajectory, therefore, the component of the goal vector is not a function, but a functional, the integrand of which, the risk vector C, depends on the state vector x and the control vector u

$$\mathbf{J(x,u)} = c \begin{bmatrix} \int_{t_0}^{t_1} C_1(\mathbf{x}, \mathbf{u}_1) dt \\ \int_{t_0}^{t_1} C_2(\mathbf{x}, \mathbf{u}_2) dt \\ \vdots \\ \int_{t_0}^{t_1} C_m(\mathbf{x}, \mathbf{u}_{m1}) dt \end{bmatrix}. \tag{13}$$

Taking into account the specifics of plotting the course, we assume that the target functional of own ship is the first component of the vector of the target functional, and for the remaining components, we accept the hypothesis of their trajectories and controls. An important feature of the problem is that all components of the target functional vector are convex, have extrema, and can be optimized.

On the other hand, all ships have the same type of linear ship dynamics model, the only difference is in the object matrices Ai and control matrices Bi. In general, these constraints have the form $dx/dt - \varphi(x, u) = 0$. Thus, we have the task of controlling a ship with a vector target function, Figure 1.

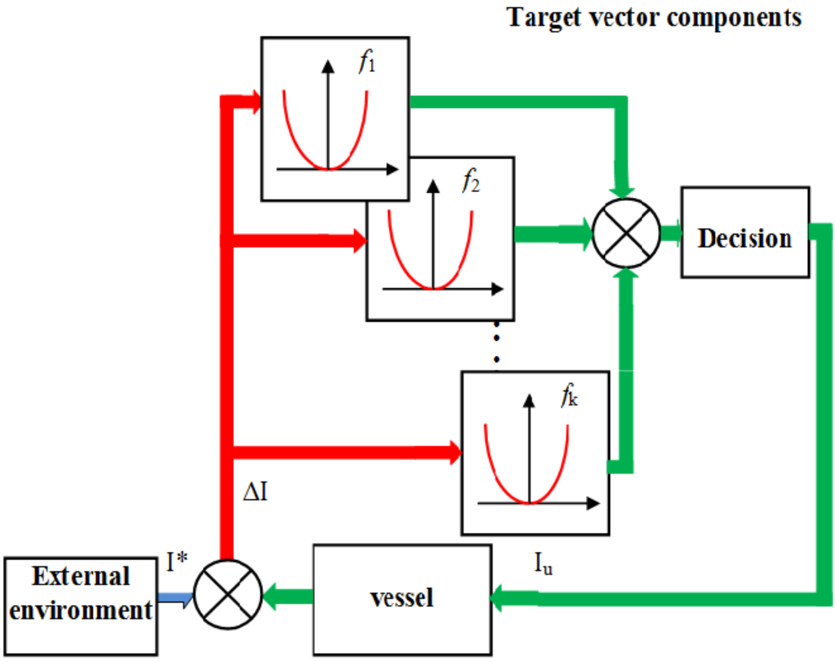

Fig. 1. Control structure with a vector goal

Since we have a very complex problem, we first consider the solution of a one-dimensional optimal control problem with a convex functional and a fully controllable dynamical system under constraints.

For a convex goal functional $F(x, \dot{x}, t)$ and constraints in the form of a dynamical system described in the Cauchy form, we have the task

$$(\mathbf{x}^*, \mathbf{u}^*, t^*) \to extr \int_{t_0}^{t_1} F(\mathbf{x}, \mathbf{u}, t)dt; \quad \dot{\mathbf{x}} - \mathbf{f}(\mathbf{x}, \mathbf{u}, t) = \mathbf{0}; \quad \left.\begin{array}{r} \mathbf{x}(t_0) = \mathbf{x} \\ \mathbf{x}(t_1) = \mathbf{x_1} \end{array}\right\}. \tag{14}$$

The resulting problem is a Lagrange problem with equality-type constraints, and we seek its solution using the Lagrange functional

$$\tilde{L} = \int_{t_0}^{t_1} [f_0\lambda_0 + \lambda^T\mathbf{f} - \lambda^T\dot{\mathbf{x}}]dt = \int_{t_0}^{t_1} L(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}, t)dt \tag{15}$$

Since the integrand of the goal function is convex and the constraints are controllable, the Kuhn - Tucker conditions are satisfied for the integrand of the Lagrange functional

$$L(\mathbf{x}^*, \dot{\mathbf{x}}, \mathbf{u}, \lambda) \le L(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}, \lambda) \le L(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}, \lambda^*) \tag{16}$$

We separate the Hamilton function in the Lagrange function

$$L(\mathbf{x}, \dot{\mathbf{x}}, \mathbf{u}, \lambda) = H(\mathbf{x}, \mathbf{u}, \lambda) - \lambda^{\mathbf{T}*}\dot{\mathbf{x}} \tag{17}$$

We express the controls in terms of the Lagrange multiplier $u = u(\lambda)$ and, considering the control as an independent variable, we obtain

$$H(\mathbf{x}^*, \mathbf{u}) - \lambda^{*\mathbf{T}}\dot{\mathbf{x}} \leq H(\mathbf{x}^*, \mathbf{u}^*) - \lambda^{*\mathbf{T}}\dot{\mathbf{x}} \leq H(\mathbf{x}, \mathbf{u}^*) - \lambda^{*\mathbf{T}}\dot{\mathbf{x}} \tag{18}$$

Eliminating similar ones, we obtain the Kuhnna - Tucker condition for the Hamilton function in the problem with a convex goal functional and controlled constraints

$$H(\mathbf{x}^*, \mathbf{u})_{|\lambda^*} \leq H(\mathbf{x}^*, \mathbf{u}^*)_{|\lambda^*} \leq H(\mathbf{x}, \mathbf{u}^*)_{|\lambda^*} \tag{19}$$

This inequality breaks down into two conditions

$$\left.\begin{array}{l} \mathbf{x}^*_{|\lambda^*} \to minH(\mathbf{x}, \mathbf{u}^*) \\ \mathbf{u}^*_{|\lambda^*} \to minH(\mathbf{x}^*, \mathbf{u}) \end{array}\right\}. \tag{20}$$

The first inequality of the system gives rise to Bellman's principle, and the second inequality gives rise to the Pontryagin maximum principle.

In the problem under consideration, we take into account the constraints associated with the dynamics of the ship

$$\mathbf{J}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \int_{\mathbf{t_0}}^{\mathbf{t_1}} \mathbf{C_1}(\mathbf{x}, \mathbf{u_1})dt \\ \int_{\mathbf{t_0}}^{\mathbf{t_1}} \mathbf{C_2}(\mathbf{x}, \mathbf{u_2})dt \\ \vdots \\ \int_{\mathbf{t_0}}^{\mathbf{t_1}} \mathbf{C_m}(\mathbf{x}, \mathbf{u_{m1}})dt \end{bmatrix} = \int_{\mathbf{t}} \mathbf{C}(\mathbf{x}, \mathbf{u})dt; \tag{21}$$

$$\varphi(\mathbf{X}, \mathbf{u}) = \begin{bmatrix} \dot{\mathbf{x}} - A_1\mathbf{x} - B_1\mathbf{u}_1 \\ \dot{\mathbf{x}} - A_2\mathbf{x} - B_2\mathbf{u_2} \\ \vdots \\ \dot{\mathbf{x}} - A_n\mathbf{x} - B_n\mathbf{u}_n \end{bmatrix} = 0;$$

Then, after supplementing the constraints with the optimality condition (5), the Hamilton vector function in this problem has the form, we emphasize in the notation vectors

$$\overrightarrow{\mathbf{H}}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{u}}) = \lambda_0\overrightarrow{\mathbf{C}}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{u}}) - \lambda(\overrightarrow{\varphi}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{u}}) - \dot{\mathbf{x}}) - \lambda\overrightarrow{\mathbf{C}}(\overrightarrow{\mathbf{x}}, \overrightarrow{\mathbf{u}}). \tag{22}$$

Therefore, if λ_0 can be considered equal to one, then Λ is a cellular matrix similar to the matrix in equation (11). Let us assume that the problem is stationary, which makes it possible to use the maximum principle. Consequently, the optimality conditions take the form

$$u^*_{|_{\lambda^*}^{x^*}} \to \begin{cases} \frac{\partial \mathbf{H}}{\partial \mathbf{x}} = +\frac{d\lambda}{dt} \\ max\mathbf{H}(\mathbf{x}^*, \mathbf{u}). \\ \frac{\partial \mathbf{H}}{\partial \mathbf{u}} = 0 \end{cases} \tag{23}$$

Since we are looking for the strong optimum of the stationary problem according to the maximum principle, the Hamilton function is constant on the optimal trajectory. Then, if the Lagrange multiplier matrix is unit $\Lambda = I$, the optimality condition (23) is satisfied.

Thus, the general divergence problem, provided that all components of the vector of integrands of the goal functional are convex, has a simple solution

$$\lambda = \begin{pmatrix} \begin{pmatrix} \lambda_{11}^{11} \cdot \lambda_{1n}^{11} \\ \cdot \;\; \cdot \;\; \cdot \\ \lambda_{n1}^{11} \cdot \lambda_{nn}^{11} \end{pmatrix} & \dots & \begin{pmatrix} \lambda_{11}^{1n} \cdot \lambda_{1n}^{1n} \\ \cdot \;\; \cdot \;\; \cdot \\ \lambda_{n1}^{1n} \cdot \lambda_{nn}^{1n} \end{pmatrix} \\ \cdot & \cdot & \cdot \\ \begin{pmatrix} \lambda_{11}^{n1} \cdot \lambda_{1n}^{n1} \\ \cdot \;\; \cdot \;\; \cdot \\ \lambda_{n1}^{n1} \cdot \lambda_{nn}^{n1} \end{pmatrix} & \dots & \begin{pmatrix} \lambda_{11}^{nn} \cdot \lambda_{1n}^{nn} \\ \cdot \;\; \cdot \;\; \cdot \\ \lambda_{n1}^{nn} \cdot \lambda_{nn}^{nn} \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} & \dots & \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} \\ \cdot & \cdot & \cdot \\ \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} & \dots & \begin{pmatrix} 1 \cdot 0 \\ \cdot \cdot \cdot \\ 0 \cdot 1 \end{pmatrix} \end{pmatrix} = I. \tag{24}$$

A simpler formulation of optimality consists in the absence of mutual sensitivity with respect to objective functions

$$\frac{\partial C_i}{\partial C_j} = 0; \quad i = \overline{1,n}; \quad j = \overline{1,n}; \quad i \neq j; \quad \frac{\partial C_i}{\partial C_i} = 1; \quad i = \overline{1,n}. \tag{25}$$

Consequently, despite the complexity, the general divergence problem has a global optimal solution determined by condition (25). This condition means that when plotting a course, the divergence distance between the "i-th" and "j-th" vessel must always ensure that the specified risk of both vessels is maintained. The target hazard ellipse of one vessel shall not intersect the target hazard ellipse of another vessel. Thus, the laying is carried out according to the criterion of minimum costs, provided that it is optimal (25). This formulation of the question is beneficial from the point of view of forecasting the actions of the courts. Since in this case each of the participants in the operation reaches the optimum, assuming the convexity of the integrands, it is practically assumed that the actions of the navigators are reasonable. For autonomous ships, where there is no "human" factor, one can always count on the "reasonableness" of the actions of the automatic control system (ACS).

4 Experiment, Results and Discussion

Modeling was carried out in MATLAB environment and on the Navi Trainer 5000 navigation simulator [1, 4]. Knowing the conditions for the optimal solution of the discrepancy problem, we can consider the algorithm for plotting a course for the automatic system. Since the ACS system is a link of the artificial intelligence of an autonomous vessel and eliminates the risks of the "human" factor, the machine moves the vessel into the risk field. Figure 2. shows a diagram of the construction of the risk field.

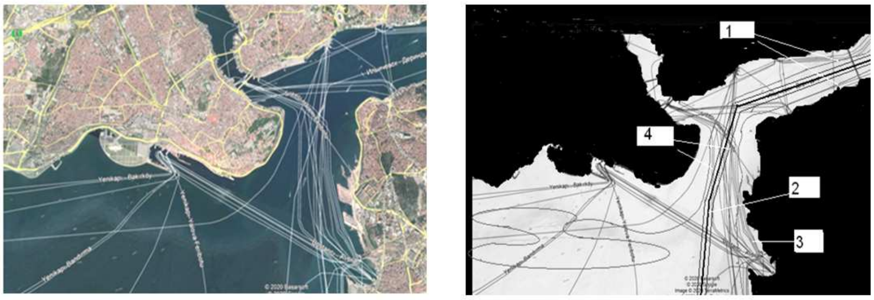

Fig. 2. Scheme for constructing the risk field

Critical risk (position 1, Figure 2) defines the unacceptable positions, and the acceptable risk (position 2, Figure 2), defines the areas with acceptable but undesirable positions and the field of specified risks (position 3, Figure 2) defines the area of the trace, position 4 The route itself is carried out for reasons of minimum costs when passing the route. In reality, there are standard solutions (algorithms) for laying a route for a route. However, the automatic system must have a criterion that determines the freedom of decision, otherwise the discrepancy problem requires the participation of a person who evaluates the risk of the decision made. Thus, the risk field ensures the “reasonableness” of the ACS action. The route has been set, the schedules have been agreed upon, but a risk field is needed to make a decision. The task of building a risk field at the modern level of Internet technologies is not difficult. A collection of electronic navigation charts (ENC) as well as hydrometeorological information of navigation areas is loaded into the network, you just need to highlight the risk level lines. Here is the question about the companies that sell electronic cards (TRANSAS, C-MAP, NOAA).

The second step in solving the route optimization problem is taking into account the environment at the current time. This is done by analyzing the schedules of the movement of ships, radar and optical fields, determining the coordinates of oncoming ships and their maneuvers. This operation can also be obtained by exchanging information with other vessels containing, in addition to standard data, the maximum risk and variance along the axes of risk distribution. To this information, you can add the ship's coordinates, speed, heading and maneuver. This simple message facilitates the task of diverging ships. While this is not the case, we will consider the radar assessment of the situation to be consistent.

Now you can perform the next step - forecasting the development of the operation. At this stage, the trajectory of own ship is adjusted, taking into account the possible risks of entry into the risk area of other ships, Figure 3.

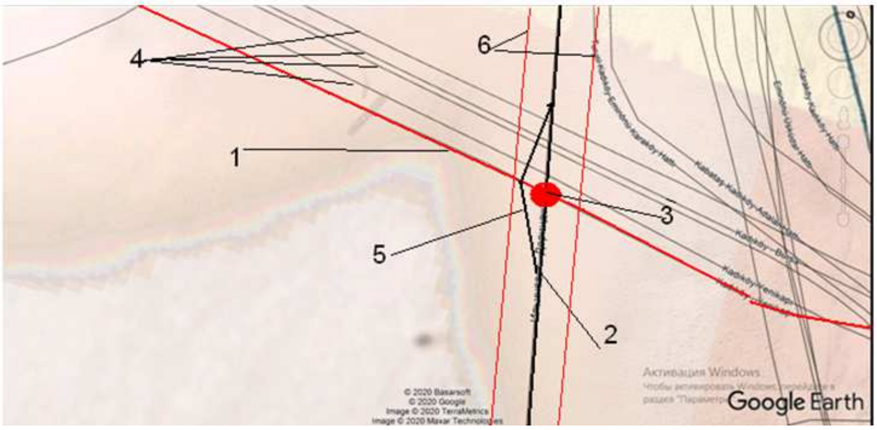

Fig. 3. Correction of your own course based on the results of the forecast of the development of the situation

For the selected trajectory 2, from the intersecting trajectories of other vessels 4, critical trajectories 1 are determined. When a critical trajectory is detected, a divergence maneuver 5 can be planned within the field of permissible risks 6. At the same time, the intersections of the trajectories are not critical if at the moment of crossing the distance between the vessels does not violate the zones of the given risk. The course correction is influenced only by critical trajectories, this is the standard plotting of the course [10–12, 16, 17] with a restriction on exiting the risk corridor 6. This operation of the course correction is current, which guarantees control over the situation. However, in spite of the optimality of the performed corrections, situations are possible when it is necessary to quickly carry out the operations of diverging vessels. In this case, either the problem of sliding along the line of equal risk of the target vessel is solved, or, in the case of movement between the vessels, a discrepancy in the minimum of the risk gradient is performed. In both cases, the course is plotted taking into account the preservation of the area of the given risk. Situations are possible when the risk increases. For example, when mooring or bunkering a vessel on the move, the risk increases, which is inevitable in the essence of the operation, and here the speed regime changes, in contrast to the divergence in the open sea, where a change in the vessel's speed is undesirable.

Figure 4 shows the optimal control algorithm in the general navigation problem.

This algorithm consists of:

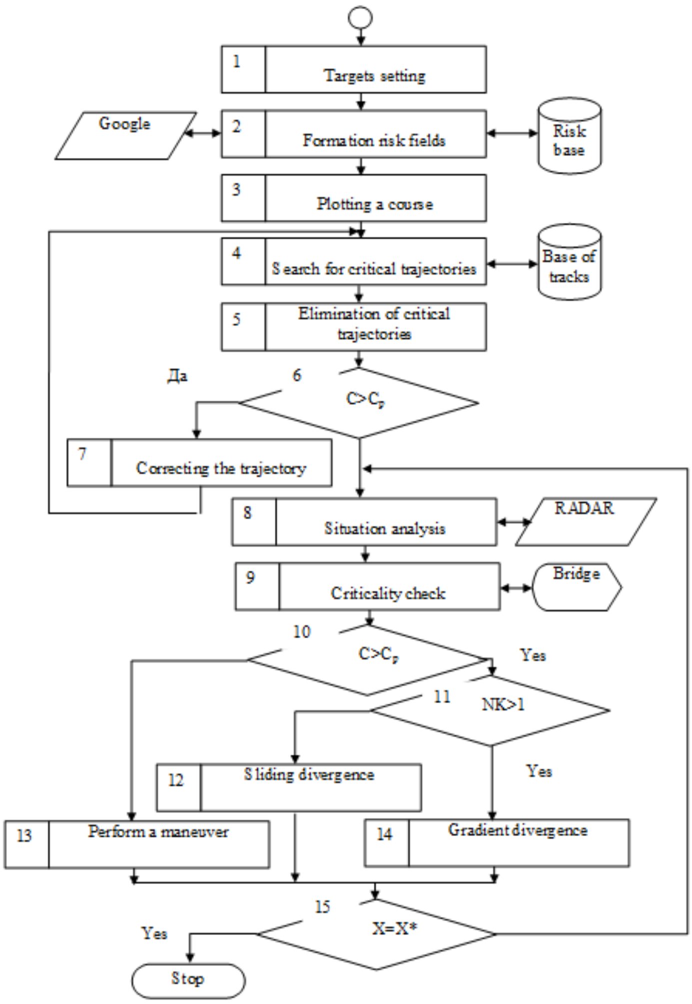

Fig. 4. Algorithm of optimal control in the general problem of navigation

- block 1 for setting goals, in which the points and times of the beginning and end of the movement are determined;
- Block 2 of the formation of the risk field, which uses the risk base and data from satellite navigation and electronic cartography;
- block 3 plotting the course according to the criterion of minimum costs. This operation does not require operator participation, as there are clear criteria and limitations;
- block 4 search for critical trajectories;
- block 5 of elimination of critical trajectories;
- block 6 for comparison of risks. If risks are found that are higher than acceptable, go to block 7;
- block 7 of correction of trajectories and graphs of their movement. If the risks are less than permissible, go to block 8 for the analysis of the situation and then to block 9 for checking the criticality;
- block 8 of the analysis of the situation, which uses the information of the radar and other radio navigation equipment;
- block 9 for checking the criticality based on the data of block 8 and visual data;
- block 10 for comparison of risks. If the risk does not exceed the permissible, go to block 13 for performing the maneuver. Otherwise, go to block 11 for analyzing the situation. If the number of vessels with a critical trajectory is one, then go to block 12 to diverge by sliding along the lines of a given risk. If the number of vessels is more than one, then go to block 14 of the gradient divergence. The second part of the algorithm (blocks 8-15) is executed continuously until the end point of the route. The principal difference of the considered algorithm is its optimality in terms of the risk criterion and compatibility with use for autonomous ships.

In Figure 5 shows the results of mathematical modeling of the processes of divergence of ships. In fig. 5a shows the divergence trajectory 5 with one vessel, built for the case of no intersection of the zones of the given risk 3, 6. In this case, the sliding trajectory 2 is repeated with an offset to the minor axis of the self-risk ellipse.

In Figure 5b shows the results of mathematical modeling of divergence processes with several vessels. In this situation, the intersection of the lines of the given risks 2 and 4 is possible. To ensure the optimal divergence, in this case, the movement is organized along the minimum of the gradient.

The issues of solving the route optimization problem using the risk criterion are considered. The problem of optimal control of a system of dynamic objects with a vector - functional was posed and solved; optimality of control is achieved due to optimization of the vector - functional on the entire trajectory of motion; optimal control problem with a vector - functional, when fulfilling the hypothesis about the convexity of the integrands of the vector – functional components, is solved using the calculus of variations; as a result of solving the optimal control problem, a simple algorithm was obtained for constructing optimal trajectories of the vessel during the avoid collision maneuver; an algorithm for constructing the optimal trajectory of the vessel's movement using risk fields has been

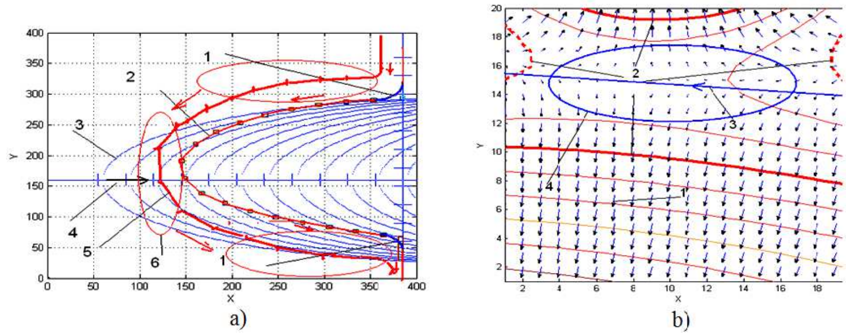

Fig. 5. Results of mathematical modeling of ship divergence processes

obtained; mathematical modeling of collision avoidance processes with one or several vessels was carried out using the risk criterion.

5 Conclusions

The paper considers and resolves the issues of constructing an optimal trajectory using the risk criterion.

The scientific novelty of the obtained results consists in the fact that for the first time a method, algorithmic and software for an automatic control system were developed that solve the problem of optimal routing and optimal divergence with one or more targets using the risk criterion. This is achieved due to the constant, with the clock cycle of the on-board controller, measuring the parameters of the vessel's movement, using them for solving the problem of minimizing the risk functional vector for constructing a field of risk levels, forming control for divergence along a given risk level line.

The practical value of the obtained results is that the developed method and algorithms are implemented in software and investigated by solving the problem in a fully automatic mode in a closed circuit with the control objects in a MATLAB environment for various types of vessels, targets, navigation areas and weather conditions. The experiment confirmed the operability and efficiency of the method, algorithmic and software, which makes it possible to recommend them for the development of mathematical support of the automatic vessel movement control systems to solve the problems of optimal routing and optimal divergence using the risk criterion.

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