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NON-HAMILTONIAN QUANTUM MECHANICS AND THE NUMERICAL RESEARCHES OF THE ATTRACTOR OF A DYNAMICAL SYSTEM.

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This article – introduction to the structural theory of general view dynamical systems, based on construction of dynamic quantum models (DQM), offered by the author. This model is simply connected with traditional model of quantum mechanics (i.e. with the Schrodinger equation). At the same time obtained thus non – Hamiltonian quantum dynamics is easier than classical one: it allow building the clear structural theory and effective algorithms of research for concrete systems. This article is devoted mainly to such task. The algorithm of search for DQM attractors, based on this approach, is offered here.

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Introduction

For numerical methods (in radio physics, chemical kinetics, biology, economics) such statement of a task is traditional and natural. On the given mathematical model, i.e. actually on the given system of the general view ordinary differential equations it is necessary to obtain its dynamics. I.e. it is required to find its stationary and transitive states and processes, its attractors, the basic numerical invariants and characteristics. At least definitions of these concepts are necessary for the decision of such problem, but thus it is necessary to have the structural theory of general view dynamical systems. However for today [1] there is no even a standard definition for an attractor, all known structural theories concern only to narrow classes of dynamical systems. Numerical researches of dynamics inevitably demands digitization on space and time, but depending on parameters of digitization results appear sharply different.

This article – introduction in the approach to a decision of this problem based on construction of dynamic quantum models (DQM), offered by the author (see [2]). From the thesis, that quantum effects are caused ineradicable by "white noise", already the certain mathematical model of quantum mechanics follows. Dynamics in it is described by the Markov cascades (with discrete time). This model for Hamiltonian systems is simply connected with traditional one (i.e. with the Schrodinger equation) and its construction can be considered, as a method of the decision of spectral tasks. But such model is not connected with Hamiltonian structure in any way, it is determined for an any ordinary differential equation on any smooth Riemann variety. Obtained thus non – Hamiltonian quantum dynamics is easier than classical one: DQM is the Markov cascade. It is well investigated structure with the clear theory, allowing to build effective algorithms of research for concrete systems. This article is devoted mainly to such task. On the other hand, at aspiration to zero of fluctuations, i.e. in quasiclassical limit, results of quantum dynamics transform into statements about corresponding classical dynamics. In such way it is possible to receive, for example, the full theory of hyperbolic dynamic systems, including the thermodynamic formalism. In such way the decision of a problem on density of nonwandering points in Anosov systems was obtain. At last, addressing to reality (according to a paradigm of programming), it is necessary to recognize, that exactly quantum (instead of classical) dynamics corresponds to the reality, which objects are connected always with unpredictability.

Definition of the dynamical quantum model (DQM).

Let $p(x)$ is a n – dimensional smooth vector field on a n – dimensional smooth Riemann variety M, where $x(x_1, x_2, \ldots, x_n) \in R^n$ are local Euclidean coordinates on M, $p_i(x) \in C^\infty(R^n)$ (i = 1, …, n) . On every phase curve $x(t) \in M$ of the dynamical system (DS), generated by this vector field

$$\frac{dx_i}{dt} = p_i(x) \qquad (i = 1, \ldots, n), \qquad (1)$$

consider the integral of the "shorten action" $s(t) = \int_{x(t)} p(x)dx = \int_0^t \|p(\tau)\|^2 d\tau$, where

$\|p(\tau)\|^2 = \sum_{i=1}^n p_i^2(\tau)$. The value $s(t)$ on each curve $x(t)$, different from a stationary point, is diffeomorphically expressed through t and refers to as " optical time ". Let ρ - such metrics, that $s(t) = \int_{x(t)} d\rho : d\rho = \|p(t)\|^2 dt$.

Further the deduction of dynamical quantum model for DS(1) (at "a physical level of strictness") follows. In result we shall obtain the formulation, which then is considered as formal definition. So, the distance d, gone from a point on a trajectory in time Δt is equal $d = \int_0^{\Delta t} \|p(\tau)\| d\tau =$

$= \|p(t_c)\| \cdot \Delta t$, where $p_c = p(t_0)$ – average value $(0 \le t_0 \le \Delta t)$. (Certainly, it under condition of unitary round of a trajectory in time Δt: points of turn are a special case). Further we assume, that fluctuations generates by "white noise" $\xi(t)$, working on configuration space with dispersion Dξ (t) = $\sigma^2 t$, where diffusion factor σ^2 is a constant on a considered time interval. Should pass some time Δt while the point will be displaced on such distance d from a starting position which will exceed the root-mean-square mistake caused $\xi(t)$ in time Δt, i.e. $\|p_c\|\Delta t$ will exceed $\sqrt{\sigma^2 \Delta t}$. At such minimal Δt $\|p_c\|\Delta t = \sigma\sqrt{\Delta t}$, whence $\sigma^2 = \|p_c\|^2 \Delta t$ and, hence,

$$\Delta t = \frac{\sigma^2}{\|p_c\|^2}, \qquad d = \|p_c\|\Delta t = \frac{\sigma^2}{\|p_c\|} \qquad (2).$$

Here, under the assumption, Δt – that minimal time interval after which there is an opportunity to make new measurement which difference from former will exceed an error, i.e. to receive significantly different measurement. Owing to (2)

$$\sigma^2 = \|p_c\|^2 \Delta t \approx \int_0^{\Delta t} \|p(\tau)\|^2 d\tau = s(\Delta t) .$$ Thus, 1) the time interval between the nearest

significant measurements is constant everywhere on a scale of optical time and is equal σ^2. (In other words the distance between them under the metrics ρ is equal σ^2).
2) For this time " white noise " $\xi(t)$ generates an ineradicable casual error which root-mean-square deviation is equal d – the distance on a trajectory between the nearest significant measurements.

So, the dynamic quantum model at first shifts each point on phase curve of the given dynamic system for optical time σ^2 (or on ρ – length σ^2), and then is displaced casually on the distance, which is not less, than length of a trajectory from initial up to a new point. The following strict definition generalizes this description. Definition of quantum model is given for any dynamical system (1) on any compact Riemann variety in M.

Let G – shift map on phase trajectories (1) for given time Δt on some set time scale. We shall consider continuous function $q(y, z) \ge 0$ $(y, z \in M)$, and

$$q(y, z) > 0 \iff \|z - Gy\| \le d(y), \quad \int_M q(y,z)dz = 1, \quad \int_M zq(y,z)dz = Gy, \qquad (3)$$

where $d(y) > 0$ – continuous function on M. Here $q(y, z)$ sets density of " the local casual dissipation caused by white noise ", numbers $d(y)$ are assumed small enough. Then

Definition 1. Dynamical quantum model (DQM) for the given dynamical system (1) we shall name the Markov process with transitive function

$$P(y, A) = \int_A q(y,z)dz \ (A \subset M).$$

Having set initial distribution, we shall obtain the Markov process P with this initial distribution and transitive function P(y, A): if μ_t is a distribution during the moment t, Δt – a time period between two nearest measurements, then DQM sets new distribution $P(\mu_t) = \mu_{t+\Delta t}$ at the moment of time $t + \Delta t$.

Thus, proceeding from the differential equation (1), we come to the discrete dynamical system with a time period at least σ^2 on a scale of optical time. At first sight step-type behavior of time in DQM can surprise: in traditional model of quantum mechanics only spatial variables errors are taken into account. But, apparently from the DQM deduction, step-type behavior of process of measurement of time is inevitable consequence from ineradicable casual errors of coordinates and impulses. Really, a clock or some other device finally is necessary for measurement of time. But as these measure indications and speeds of there changes are determined inexactly, then also time is known only with some error.

DQM attractors.

Attractor is the key concept of the theory of dynamical systems. Physical sense of attractor is that it " space of the established modes ". The point of phase space contains in attractor if it belongs to the carrier of " a stationary states of system ", i.e. belongs to a measure, which are not varying in due course. In simple traditional examples attractor represents the union of final number of stationary points of phase space and the closed curves on which there are cyclic processes. However the main interest represents difficultly arranged so-called "strange attractors". The most known such example is the three-dimensional attractor of Lorentz's system, used in meteorology. It have fractional dimension with extremely complex turbulent dynamics, substantially explaining difficulty of weather forecasting.
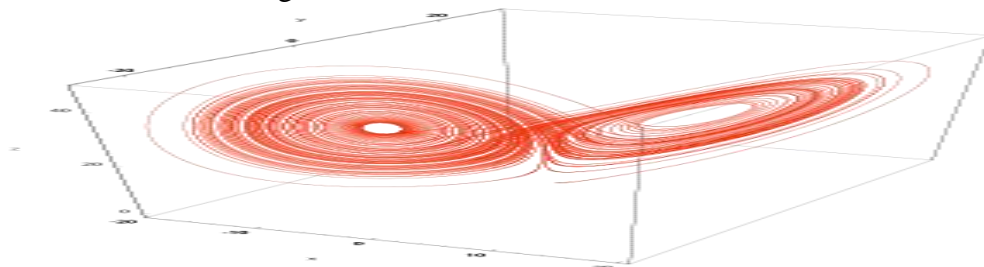
Important that for any final Markov chain attractor is determined unequivocally. At aspiration to zero of diameter of cells – the states of a chain – attractors of Markov chains evenly converge to the Markov cascade attractor. Formally specified approximation and estimations of convergence are described by the following statements.

Definition 2. Let phase space M is compact; P is set DQM on M. Stationary (equilibrium) state of DQM is a probability measure μ on M if $P\mu = \mu$. A DQM attractor is a union of carriers of all stationary states of DQM.

Theorem 1 (the Perron – Frobenius theorem for DQM). Let $\Lambda \subseteq M$ is an invariant set of DQM P, which is not containing nonempty own invariant subsets. Then there is a unique stationary set μ, which carrier is Λ. A state μ is ergodic. For any DQM state (probability measure) ν on Λ

$$\lim_{n\to\infty} \sum_{k=1}^{n} P^n \nu = \mu.$$

Obviously, there is only final number invariant DQM sets Λ_k on M, which are not containing nonempty own invariant subsets. Any stationary DQM state on M is a convex combination of stationary states μ_k on Λ_k. So, DQM attractor is determined unequivocally. It can be found algorithmically since DQM – a Markov cascade – can be approximated by Markov chains.

Definition 3. Let Δ_i are cells in diameter ε of some phase space splitting for the given dynamic system. Then an ε - digitization of DQM with transitive function $P(y, A)$ and an initial state μ_0 is a Markov chain with initial values $p_i = \mu_0(\Delta_i)$ and with probabilities of transition from Δ_i in Δ_j equal $p_{ij} = \dfrac{1}{\mu_0(\Delta_i)} \displaystyle\int_{y \in \Delta_i} P(y, \Delta_j)\, d\mu_0$.

Theorem 2. For all enough small ε DQM – distribution μ_t at the moment t and distribution of its ε – digitization μ^Δ_t at this time are differ from each other only on size of the order ε: $\left| \varphi(\mu_t) - \varphi(\mu^\Delta_t) \right| < A\varepsilon \quad (0 \leq t < \infty)$, where φ – "observable", i.e. the continuous limited function on M, A is a constant of DQM.

Thus, DQM attractor is determined unequivocally. As well as for final Markov chain this attractor consists, generally speaking, of several not intersected invariant (basic) subsets. And basic sets consists, generally speaking, of several not intersected connected subsets, that DQM transpose cyclically. It is possible to show, that exactly this cyclic structure explains the spin phenomenon in quantum mechanics [3].

Algorithm of search for DQM attractors.

Let phase space is split into cells Δ_i by diameter ε/A, where A is a constant from the theorem 2. Digitization of the Markov cascade induces symbolical dynamics on set of cells Δ_i, more precisely speaking, dynamics of topological Markov chain, for which cells Δ_i are states. Obtained thus discrete dynamics has the clear structural theory and good algorithms for concrete systems, and that's all passes in DQM.

Let's enter the transitive relation of the partial order on collections of states { Δ_i }: $\Delta_i \prec \Delta_j$, if there is some trajectory of symbolical dynamics from Δ_i in Δ_j. A state Δ_i is returnable if $\Delta_i \prec \Delta_i$. Returnable states are broken into classes of equivalence: $\Delta_i \sim \Delta_j \Leftrightarrow \Delta_i \prec \Delta_j \prec \Delta_i$. Then DQM attractor with accuracy ε is consist of classes of equivalence of returnable states. So, if $\Omega = \{\Delta_i\}$ is all phase space of ε – digitization H of DQM, i.e. collection of all cells in diameter ε/A, then $H(\Omega) \supset H^2(\Omega) \supset H^3(\Omega) \supset \ldots \supset H^n(\Omega)$. And if $H^n(\Omega) = H^{n+1}(\Omega)$, then $H^n(\Omega)$ is an attractor of ε – digitization H, i.e. it's DQM attractor itself with accuracy ε for enough small ε. The following algorithm is based on these reasons.

Here set $\Omega = \{\Delta_i\}$ is defined by two following collections.

Collection 1:
1. Number of a cell – CNC (i.e. for a cell Δ_i CNC = i).
2. Quantity of cells in which it is possible to get from cell CNC on some trajectory of symbolical dynamics for one step – CIQ.
3. The list of such cells – ICL.
4. Number of first of them, yet not considered by program – NCN.

Number of cell CNC in the following file 2, putting to a cell in conformity its class of equivalence of returnable stations – MN.

Collection 2:
1. Number of a cell in the collection – MN.
2. Number of a class of this cell – KN.
3. Number of this cell in the collection 1 – CNC.

Besides the following switches are used: 1) CN – current number of a considered cell; 2) the total number of the cells already considered by the program – TN; 3) current number of a class considered by the program – CK. Then algorithm of the program on the set collection 1 receives in

collection 2 some class of equivalence of the returnable stations for which value KN is maximum. This class is the DQM attractor itself with accuracy ε.

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                    ┌─────────────────────────────┐
        ◁───────────│   Input of a collection 1   │───────────▷
                    └─────────────────────────────┘
                                 │
                    ┌─────────────────────────────┐
                    │  NCN = 1, MN = 0, KN = 0    │
                    └─────────────────────────────┘
                                 │
                    ┌─────────────────────────────┐
                    │    CN =1, TN =0, CK = 0     │
                    └─────────────────────────────┘
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Basis cycle

NCN ≤ CIQ?   no → CN = CN − 1

In collection 1
NCN = NCN + 1

yes

KN = CK ?   MN = 0 ?   KN = CK ?

yes   no   yes   no

CK = KN
In cells of collection 2 with
numbers from MN to TN
KN = CK

TN = TN + 1

CK = CK + 1

End

MN = TN, KN = CK,
CNC from collection 2 = CNC from collection 1

NCN = NCN + 1

CN = TN + 1
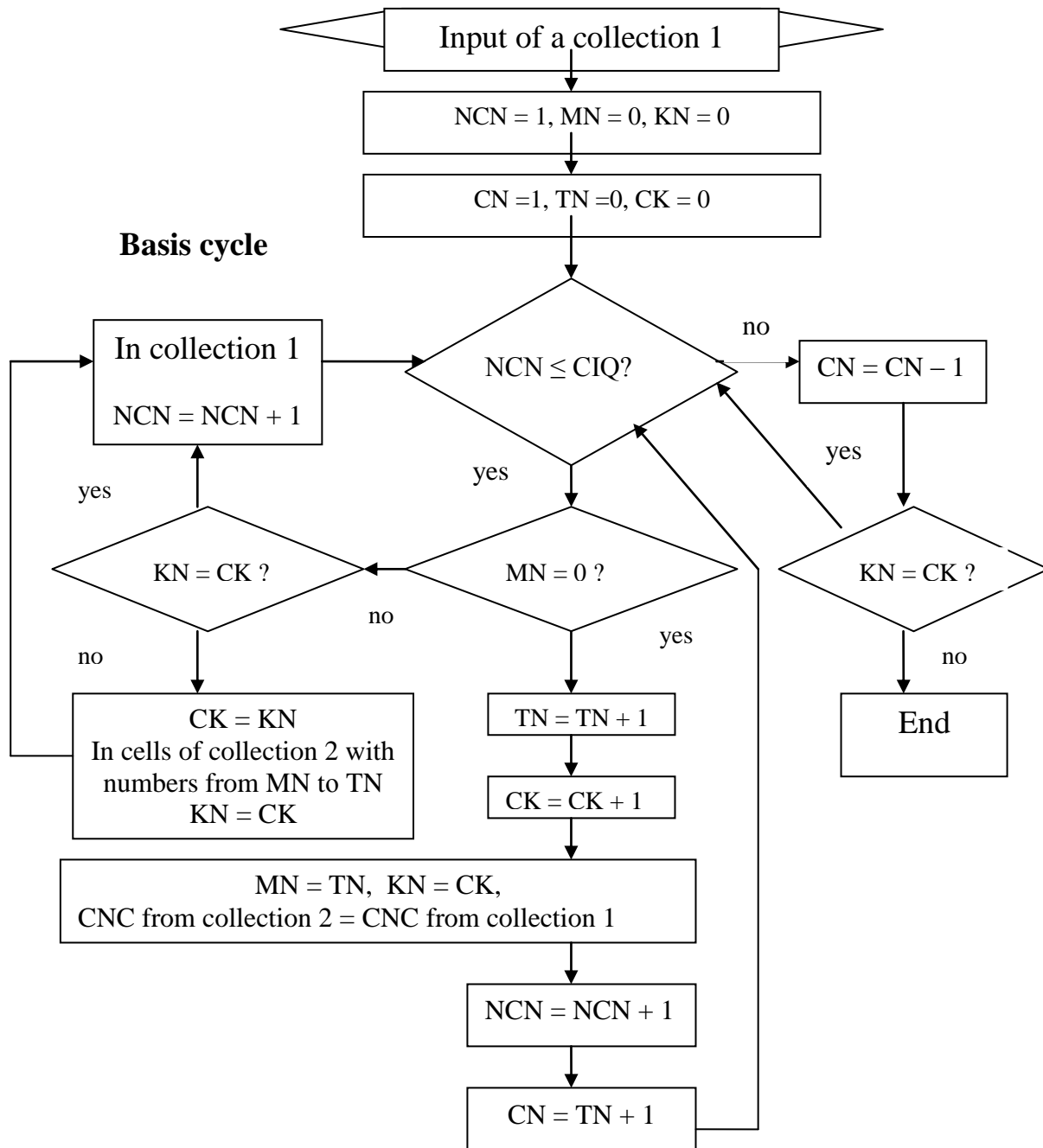
Rating of efficiency of the algorithm. Number of operations C for such algorithm simply proportionally M − the number of cells in image H(Ω). So C ≤ k M ≤ k N, where N - number of all cells in phase space. Thus the algorithm has linear complexity; the factor of proportionality k is rather small also.

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