

Properties of sets of solutions of equations containing fractal functions

O. V. Kotova

Kherson State University
Olga-Kotova@ukr.net

Let $\Delta_{\alpha_1(x)\dots\alpha_k(x)}^s$ be a formal s -adic representation for some number $x \in [0; 1]$, $\alpha_i = \alpha_i(x) \in \{0, 1, \dots, s-1\} = A$, i.e., $x = \sum_{m=1}^{\infty} s^{-m} \alpha_m$.

If limit $\lim_{n \rightarrow \infty} \frac{N_i(x, n)}{n}$ exists, where $i \in A$, $N_i(x, n) = \#\{k : \alpha_k(x) = i, k \leq n\}$, then its value $v_i^s(x)$ is called a frequency of digit i in s -adic representation of number x .

Theorem 1. Equation $v_i^s(x) = x$, $i \in A$, does not have rational solutions in s -adic numeral system except $x = 0$.

Theorem 2. Number $x = \frac{\gamma_1}{s} + \frac{\gamma_2}{s^2} + \dots + \frac{\gamma_k}{s^k} + \sum_{j=1}^{\infty} \sum_{l=1}^{e_j} \frac{i \cdot \beta_{lj} + i_{lj} \cdot (1 - \beta_{lj})}{s^{s_j+l}}$, where

$$\begin{aligned} \gamma_{lj} &\in A, i_{lj} \in A, k \in N, i_{lj} = i, \frac{\gamma_1}{s} + \dots + \frac{\gamma_k}{s^k}, \\ \beta_{ln} &= [(s_n + l)x_n] - [(s_n + l - 1)x_n], \\ x_n &= \frac{\gamma_1}{s} + \frac{\gamma_2}{s^2} + \dots + \frac{\gamma_k}{s^k} + \sum_{j=1}^{\infty} \sum_{l=1}^{e_j} \frac{i \cdot \beta_{lj} + i_{lj} \cdot (1 - \beta_{lj})}{s^{s_j+l}}, \\ s_1 &= k, s_n = s^{s_{n-1}}, e_n = s_{n+1} - s_n = s^{s_n} - s_n, \end{aligned}$$

is a solution of equation $v_i^s(x) = x$.

Theorem 3. For any finite sequence $\{\varepsilon_n\}$ of zeros and ones there exist $\gamma_{2(k-1)s+1}, \gamma_{2(k-1)s+2}, \dots, \gamma_{2ks-1}$, $k = 1, 2, \dots$ such that number

$$x = \Delta_{\gamma_1 \dots \gamma_{2s-1} \varepsilon_1 \gamma_{2s+1} \dots \gamma_{4s-1} \varepsilon_2 \dots \gamma_{2(k-1)s+1} \dots \gamma_{2ks-1} \varepsilon_k \dots}^s,$$

is a solution of equation $v_i^s(x) = x$. Hausdorff-Besicovitch dimension of the set K_s of solutions of equation $v_i^s(x) = x$ obtained by this algorithm is equal to $\frac{1}{2s} \log 2$.

Theorem 4. Hausdorff-Besicovitch dimension of the set E_s of all solutions of the set $v_i^s(x) = x$ is not less than $\frac{1}{2s} \log 2$.

1. Котова О.В. Континуальність множини розв'язків одного класу рівнянь, які містять функцію частоти тріскових цифр числа // Укр. мат. журн. – 2008. – т.60, № 10. – С. 1414 – 1421.
2. Працьовитий М.В. Фрактальний підхід у дослідженнях сингулярних розподілів. – Київ: Вид-во НПУ імені М.П. Драгоманова, 1998. – 296 с.

