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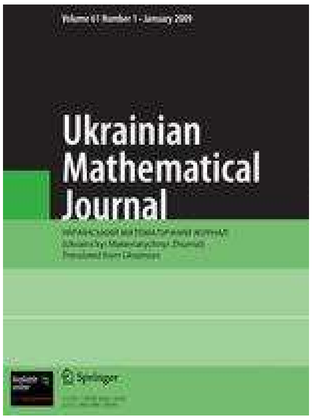
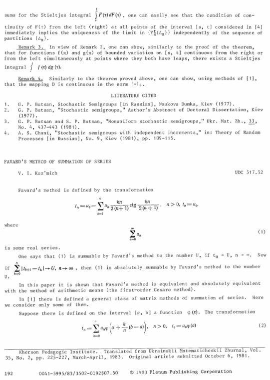
Ukrainian Mathematical Journal
March–April, 1983, Volume 35, Issue 2, pp 192-194

Favard's method of summation of series

- V. I. Kuz'mich

Translated from Ukrainskii Matematicheskii Zhurnal, Vol. 35, No. 2, pp. 225–227, March–April, 1983.





sums for the Stieltjes integral $\int_s^t F(\tau)\,dF(\tau)$, one can easily see that the condition of continuity of $F(\tau)$ from the left (right) at all points of the interval $[s, t]$ considered in [4] immediately implies the uniqueness of the limit in $\{Y_s^t(\Delta_n)\}$ independently of the sequence of partitions $\{\Delta_n\}$.

Remark 3. In view of Remark 2, one can show, similarly to the proof of the theorem, that for functions $f(x)$ and $g(x)$ of bounded variation on $[s, t]$ continuous from the right or from the left simultaneously at points where they both have leaps, there exists a Stieltjes integral $\int_s^t f(\tau)\,dg(\tau)$.

Remark 4. Similarly to the theorem proved above, one can show, using methods of [1], that the mapping D is continuous in the norm $|\cdot|_6$.

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FAVARD'S METHOD OF SUMMATION OF SERIES

V. I. Kuz'mich UDC 517.52

Favard's method is defined by the transformation

$$t_n = u_0 - \sum_{k=1}^{n} u_k \frac{k\pi}{2(n+1)} \operatorname{ctg} \frac{k\pi}{2(n+1)}, \quad n > 0, \ t_0 = u_0,$$

where

$$\sum_{n=0}^{\infty} u_n \tag{1}$$

is some real series.

One says that (1) is summable by Favard's method to the number U, if $t_n \to U$, $n \to \infty$. Now if $\sum_{k=0}^{n} |t_{k+1} - t_k| \to U$, $n \to \infty$, then (1) is absolutely summable by Favard's method to the number U.

In this paper it is shown that Favard's method is equivalent and absolutely equivalent with the method of arithmetic means (the first-order Cesaro method).

In [1] there is defined a general class of matrix methods of summation of series. Here we consider only some of them.

Suppose there is defined on the interval $[a, b]$ a function $\varphi(x)$. The transformation

$$t_n = \sum_{k=0}^{n} u_k \varphi\left(a + \frac{k}{n}(b - a)\right), \quad n > 0, \quad t_0 = u_0 \varphi(a) \tag{2}$$

Kherson Pedagogic Institute. Translated from Ukrainskii Matematicheskii Zhurnal, Vol. 35, No. 2, pp. 225-227, March-April, 1983. Original article submitted October 6, 1981.

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Suppose there is defined on the interval [a, b] a function $\varphi(x)$. The transformation

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defines a method of summation, which we denote by $\Phi[(b-a)/n]$. Special cases of it are: Favard's method for $a = 0$, $b = \pi/2$, $\varphi(x) = x\cot x$; Rogosinsky's method for $a = 0$, $b = \pi/2$, $\varphi(x) = \cos x$; the method of arithmetic means for $a = 0$, $b = 1$, $\varphi(x) = 1 - x$.

LEMMA 1. If the lower triangular regular matrix $A = (a_{nk})$ of a transformation of a sequence into a sequence satisfies the conditions: 1) $a_{nn} = O(1/n)$, 2) $a_{nk} \leqslant a_{n,k+1}$, $0 \leqslant k < n$ for $n > n_0$, then the method of summation defined by this matrix is equivalent with the method of arithmetic means and they are compatible [2].

LEMMA 2. If the normal absolutely regular matrix $B = (b_{nk})$ of a transformation of series into series satisfies the conditions: 1) $b_{nn} = O(1/n)$, 2) $b_{nk}/k \leqslant b_{n,k+1}/(k+1)$, $k_0 \leqslant k < n$, then the method of summation defined by this matrix is absolutely equivalent with the method of arithmetic means and they are absolutely compatible.

Lemma 2 is a simple consequence of Theorem 2 of [3].

THEOREM. If the function $\varphi(x)$ has a finite second derivative everywhere in $[a, b]$, and $\varphi''(x) \leqslant 0$, $\varphi(a) = 1$, $\varphi(b) = 0$, then the method $\Phi(b-a)/n$ is equivalent and absolutely equivalent with the method of arithmetic means. Moreover, they are compatible and absolutely compatible.

Proof. First we shall show that the method indicated satisfies the conditions of Lemma 1.

From the conditions of the theorem it follows that $\varphi(x)$ has bounded first derivative everywhere on $[a, b]$ and hence satisfies a Lipschitz condition. Since $\varphi(a) = 1$, this implies the regularity of the method $\Phi(b-a)/n$ [1]. Moreover,

$$\sup_{x\in[a,b]} \frac{\varphi(x)}{b-x} = \sup_{x\in[a,b]} \left| \frac{\varphi(b)-\varphi(x)}{b-x} \right| < \infty. \tag{3}$$

For the method $\Phi(b-a)/n$ $a_{nk} = \varphi\left(a + \frac{k}{n+1}(b-a)\right) - \varphi\left(a + \frac{k+1}{n+1}(b-a)\right)$, $0 \leqslant k \leqslant n$, $a_{nk} = 0$, $0 \leqslant n < k$.

Whence, using (3), we get

$$a_{nn} = \varphi\left(a + \frac{n}{n+1}(b-a)\right) = \frac{b-a}{n+1} \frac{\varphi\left(a + \frac{n}{n+1}(b-a)\right)}{b - \left(a + \frac{n}{n+1}(b-a)\right)} = O(1/n),$$

i.e., condition 1) of Lemma 1 holds.

Further, we have $a_{nk} - a_{n,k+1} = \varphi\left(a + \frac{k}{n+1}(b-a)\right) - 2\varphi\left(a + \frac{k+1}{n+1}(b-a)\right) + \varphi\left(a + \frac{k+2}{n+1}(b-a)\right)$,

$0 \leqslant k < n$, $n > n_0$. It follows from the conditions of the theorem that $\varphi(x)$ is a function that is convex upward on $[a, b]$, so $a_{nk} - a_{n,k+1} \leqslant 0$, $0 \leqslant k < n$, $n > n_0$, and condition 2) of Lemma 1 holds. Hence the method $\Phi(b-a)/n$ is equivalent with the method of arithmetic means and they are compatible.

We shall show that the method $\Phi(b-a)/n$ satisfies the conditions of Lemma 2. For it $b_{nk} = \varphi\left(a + \frac{k}{n+1}(b-a)\right) - \varphi\left(a + \frac{k}{n}(b-a)\right)$, $0 \leqslant k \leqslant n$, $n > 0$, $b_{00} = \varphi(a) = 1$, $b_{nk} = 0$, $0 \leqslant n < k$.

In order that the matrix $B = (b_{nk})$ be absolutely regular, by a familiar theorem of Knopp–Lorentz it is necessary and sufficient that one have the following conditions:

a) $\sum_{n=k}^{\infty} |b_{nk}| = O(1)$, b) $\sum_{n=k}^{\infty} b_{nk} = 1$, $k \geqslant 0$.

Since $\varphi(x)$ satisfies a Lipschitz condition on the interval $[a, b]$, one has

$$|b_{nk}| \leqslant M\left(a + \frac{k}{n}(b-a) - a - \frac{k}{n+1}(b-a)\right) = M(b-a)\frac{k}{n(n+1)}, \tag{4}$$

where M is a constant, independent of n and k. It follows from (4) that condition a) holds. Moreover,